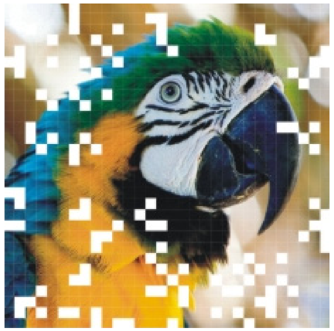
Fitting Surfaces to Polygonal Meshes using Parametric Pseudo-Manifolds

Tutorial 3



SIBGRAPI 2008

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Introduction

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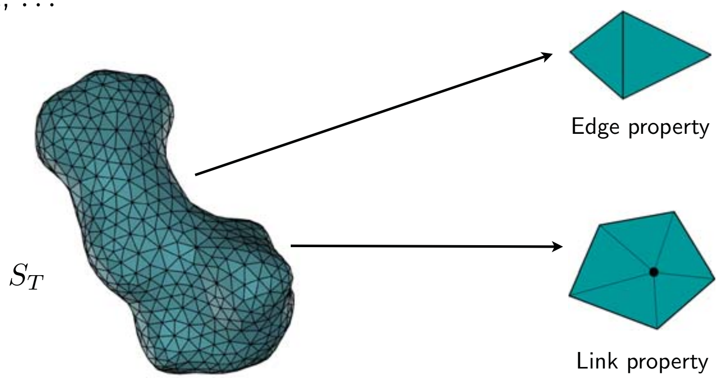
Outline

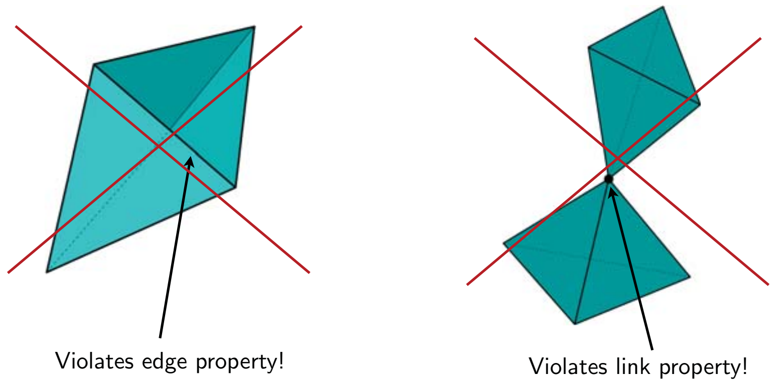
- The Surface Fitting Problem
- Traditional Approaches
- The Manifold-Based Approach
- What's Next?

The Surface Fitting Problem

We are a given a piecewise-linear surface, S_T, in $\mathbb{R}^3$, with an empty boundary, a positive integer k, and a positive number ϵ, ...



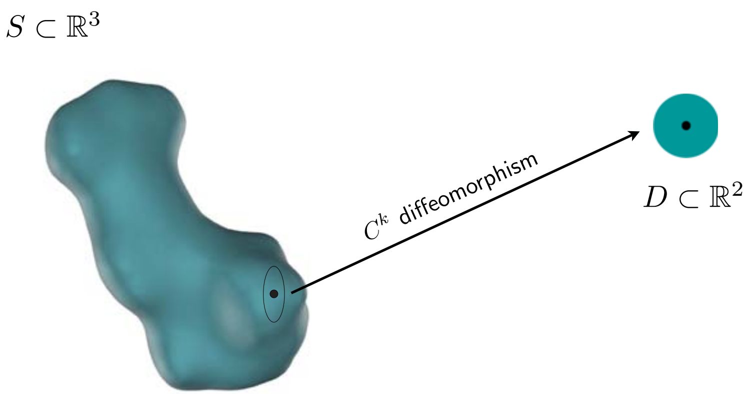
The Surface Fitting Problem



They are NOT piecewise-linear surfaces

The Surface Fitting Problem

We want to find a C^k surface $S \subset \mathbb{R}^3$ ...



The Surface Fitting Problem

such that there exists a homeomorphism, $h : S \rightarrow |S_T|$, satisfying

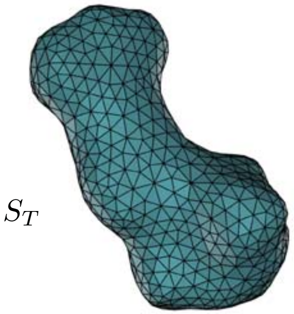
$$\|h(v) - v\| \leq \epsilon \,,$$

for every vertex v of S_T.

Topological and geometric guarantees!

The Surface Fitting Problem

From now on, we will refer to S_T as a **polygonal mesh**.



S_T

The Surface Fitting Problem

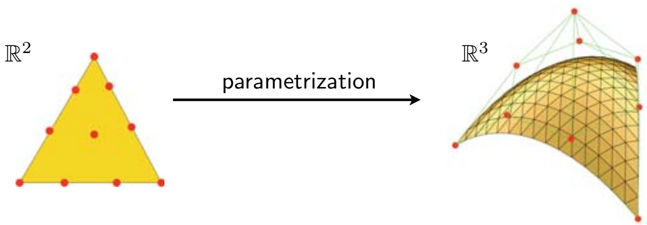
- It is a well-known and fundamental problem in CAGD.
- Reasonably well-solved for $k = 1, 2$, **but not higher**.
- Higher values of k are desirable in many applications.

Traditional Approaches

The most popular approach is certainly the parametric surface one.

Key idea:

- Assign a parametric patch to each triangle of S_T.



Traditional Approaches

and

- stitch the patches together along their common edges and vertices.

S_T S

Continuity is enforced by control point placement!

Traditional Approaches

There are several drawbacks with this approach:

- The degree d of the patches depends on k and grows rapidly with it.
- Large values of d yield surfaces of poor visual quality.
- The larger d is, the larger the number of control points.

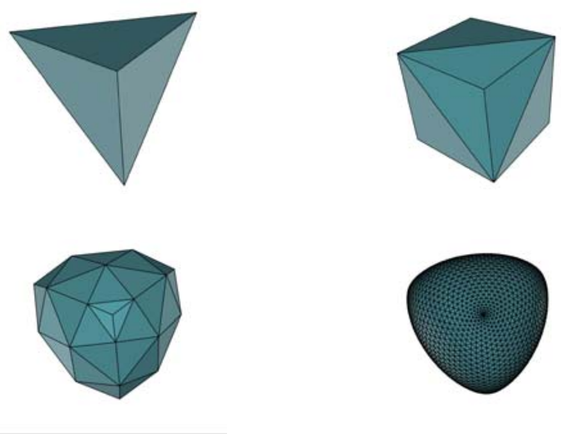
Traditional Approaches

- The larger d is, the larger the number of control points and the more difficult the problem of control point placement.
- Local control of geometry is not very flexible.

[Loop and DeRose, 1989], [Seidel, 1994], [Prautzsch, 1997], and [Reif, 1998] give C^k parametric approaches for arbitrary k.

Traditional Approaches

Another popular approach consists of using subdivision surfaces.



Traditional Approaches

Subdivision surfaces are probably the easiest and more intuitive solution for the problem **whenever the smoothness degree, k, is not large**.

For large values of k, the few existing schemes are rather complex.

See [Warren, 2002].

Traditional Approaches

Implicit surfaces can also be used to solve the problem.

They can naturally define C^∞ surfaces.

In general, the fitting problem is made into an interpolation problem.

Then, one can use RBF, MPU, moving least squares, etc.

Traditional Approaches

The main drawback of this implict surface-based approach is that the topological condition becomes a lot harder to satisfy.

More recent results **might** overcome this difficulty.

See [Shen, O'Brien, and Shewchuk, 2004] and [Kolluri, 2005].

Implicit and parametric surfaces have complementary features.

The Manifold-Based Approach

An often neglected approach, the **manifold-based** one, has the potential to easily produce C^k surfaces, for an arbitrary k (including $k = \infty$).

The manifold approach has also some advantages over the traditional approaches when it comes to certain applications, such as texture synthesis and the solution of equations on surfaces.

The Manifold-Based Approach

Here, we

- describe the manifold-based approach for the surface fitting problem,
- review the main existing solutions and their limitations, and
- point out some applications and research challenges in Computer Graphics, Image Processing, and Computer Vision that can be more naturally tackled by using manifolds.

What's Next?

II. Manifolds

III. Constructing Manifolds

IV. Fitting Surfaces to Polygonal Meshes – Part I

Coffee break

What's Next?

V. Fitting Surfaces to Polygonal Meshes – Part II

VI. Adaptive Manifold Fitting – Part I

V. Adaptive Manifold Fitting – Part II

VIII. Applications of Manifolds and Research Challenges

Manifolds

Jean Gallier
UPenn

Outline

- Manifolds: Brief History
- Informal definition
- Formal definition
- Examples
  - The Sphere
  - Real Projective Space
- Conclusions

Origins of Manifolds

- Around 1860, Mobius, Jordan, and Dyck studied the topology of surfaces.
- In a famous paper published in 1888, Dyck already uses the term **manifold** (in German).
- In the early 1900's, Dehn, Heegaard, Veblen and Alexander routinely used the term **manifold**.
- Hermann Weyl was among the first to give a rigorous definition (1913).

Keys Contributors to the notion of manifold:

Georg Friedrich Bernhard Riemann
1826-1866

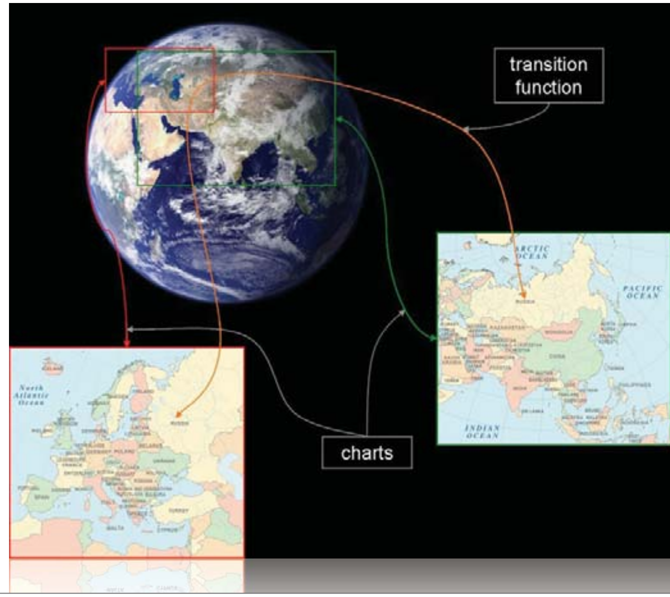
Hermann Klaus Hugo Weyl
1885-1955

Keys Contributors to the notion of manifold:

Hermann Weyl (again)

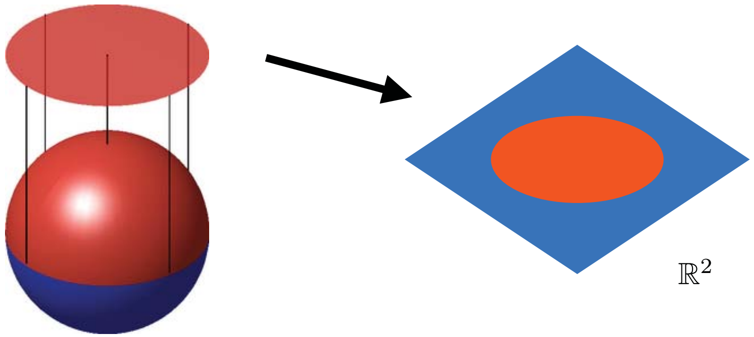
Hassler Whitney
1907-1989

Manifold: An Intuitive Picture



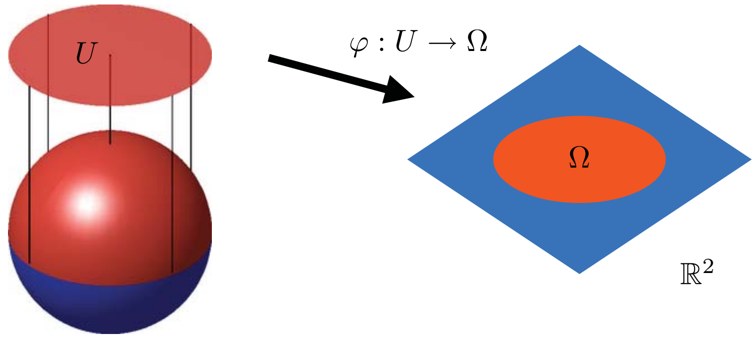
Manifolds: Informal Definition

- A manifold is a topological space with an open cover so that every open set in this cover "looks" like an open subset of $\mathbb{R}^n$.



Manifolds: Informal Definition

- To make our informal notion precise, we use homeomorphisms, $\varphi : U \to \Omega$, where $\Omega \subseteq \mathbb{R}^n$ is an open subset of $\mathbb{R}^n$.



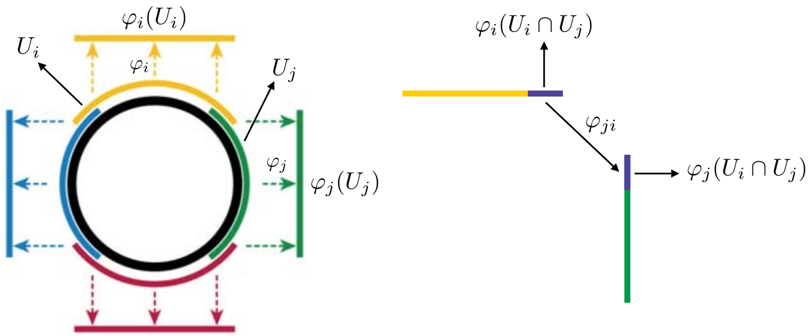
Manifolds: Informal Definition

- We also want to be able "to do calculus" on our manifolds. For this we need some conditions on **overlaps** of open sets.

Manifolds: Informal Definition

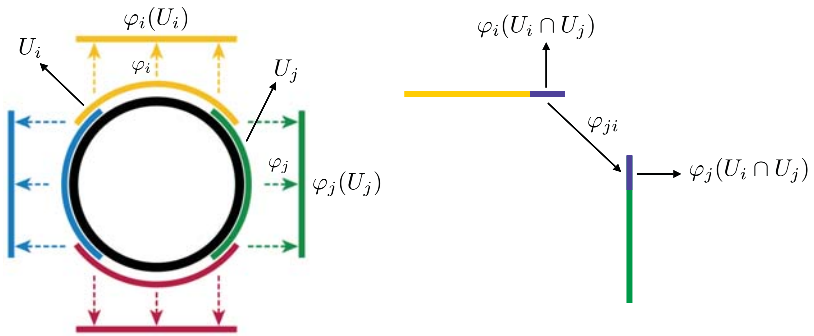
- Whenever $U_i \cap U_j \neq \emptyset$, we need some condition on the **transition function**,

$$\varphi_{ji} = \varphi_j \circ \varphi_i^{-1} : \varphi_i(U_i \cap U_j) \to \varphi_j(U_i \cap U_j)\,.$$



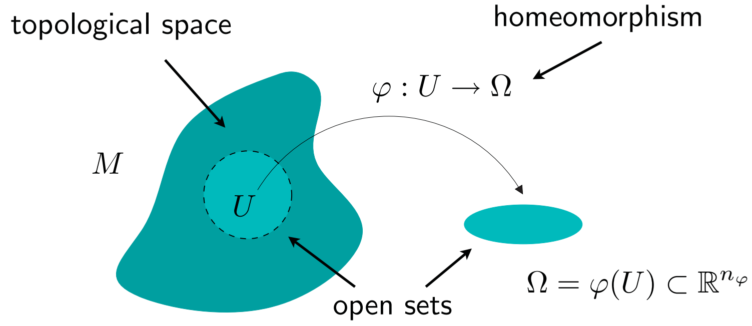
Manifolds: Informal Definition

- This is a map between two open subsets of $\mathbb{R}^n$ and we require it possess a certain amount of **smoothness**.



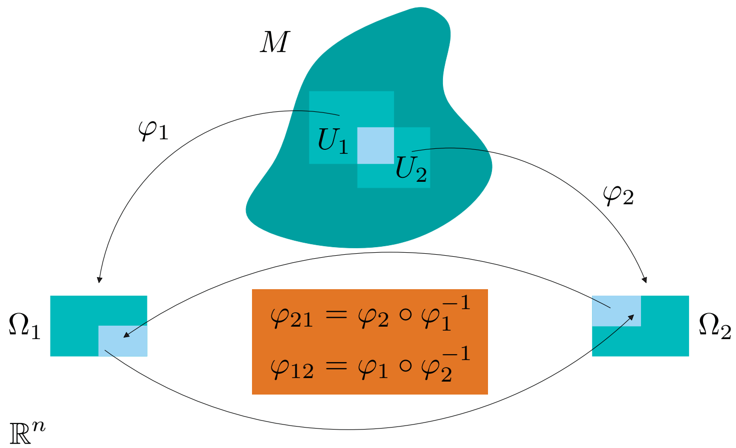
Manifolds: Formal Definition

Recall the definition of a manifold...



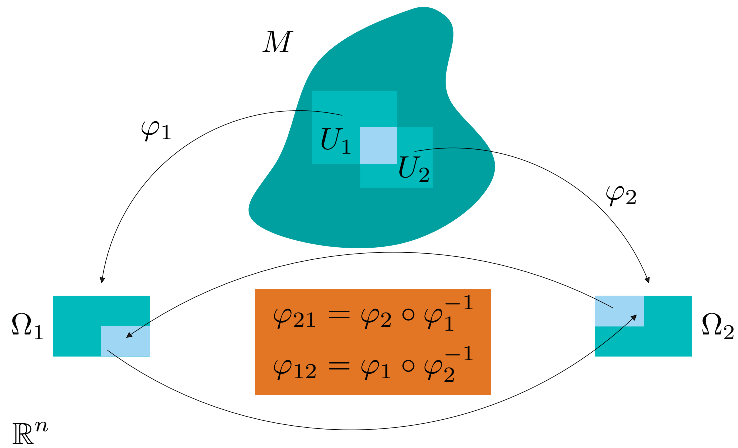
(U, φ) is called a **chart**.

Manifolds: Formal Definition



$$\varphi_{21} : \varphi_1(U_1 \cap U_2) \to \varphi_2(U_1 \cap U_2)$$
$$\varphi_{12} : \varphi_2(U_1 \cap U_2) \to \varphi_1(U_1 \cap U_2)$$

Manifolds: Formal Definition



φ_{21} and φ_{12} are the **transition functions**.

Manifolds: Formal Definition

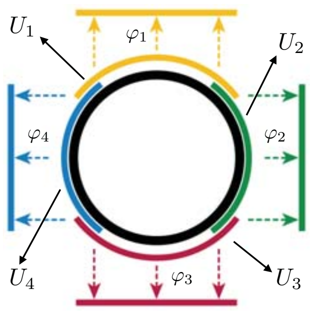
A C^k n-**atlas** is a family of charts, $\{(U_i, \varphi_i)\}_{(i \in I)}$, where I is a non-empty countable set, and such that the following conditions hold:

(1) $\varphi_i(U_i) \subseteq \mathbb{R}^n$, for all i.

(2) $M = \bigcup_{i \in I} U_i$.

(3) Whenever $U_i \cap U_j \neq \emptyset$, the transition function φ_{ji} (resp. φ_{ij}) is a C^k diffeomorphism.

Manifolds: Formal Definition



$M = \bigcup_{i=1}^{4} U_i$

φ_i is a C^k diffeomorphism

Atlas: $\{(U_1, \varphi_1), (U_2, \varphi_2), (U_3, \varphi_3), (U_4, \varphi_4)\}$

Manifolds: Formal Definition

The existence of a C^k atlas on a topological space, M, is sufficient to establish that M is an n-dimensional C^k manifold, but...

- there may be many choice of atlases;
- we get around this problem by defining a notion of atlas compatibility;
- this notion induces an equivalence relation of atlases on M;
- the set of all charts compatible with a given atlas is a maximum atlas in its class.

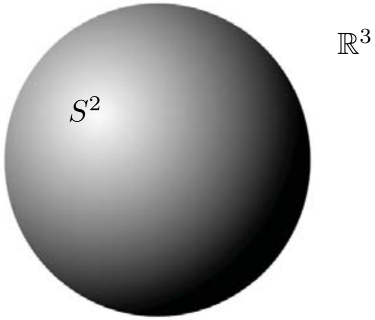
Manifolds: Formal Definition

To avoid pathological cases and to ensure that a manifold is always embeddable in $\mathbb{R}^n$, for some $n \geq 1$, we further require that the topology of M be **Hausdorff** and **second-countable**.

Examples

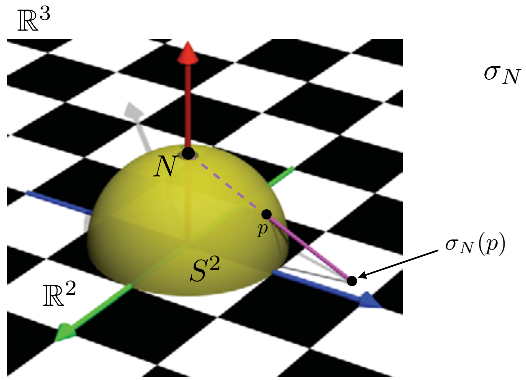
- The sphere

$$S^{n-1} = \{(x_1, \ldots, x_n) \in \mathbb{R}^n \mid x_1^2 + \cdots + x_n^2 = 1\}.$$



Examples

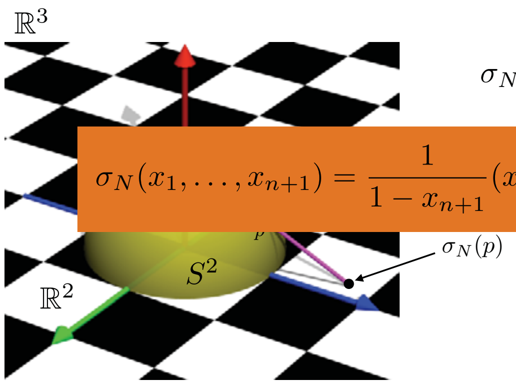
- We use stereographic projection from the north pole . . .



$$\sigma_N : S^n - \{N\} \longrightarrow \mathbb{R}^n$$

Examples

- We use stereographic projection from the north pole . . .

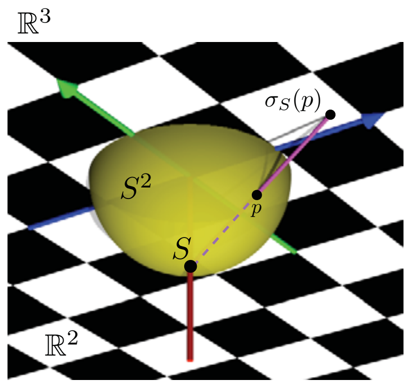


$$\sigma_N : S^n - \{N\} \longrightarrow \mathbb{R}^n$$

$$\sigma_N(x_1, \ldots, x_{n+1}) = \frac{1}{1 - x_{n+1}}(x_1, \ldots, x_n)$$

Examples

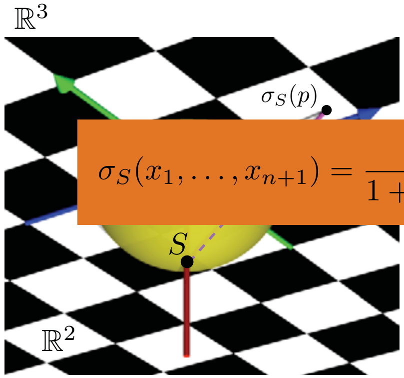
and from the south pole:



$$\sigma_S : S^n - \{S\} \longrightarrow \mathbb{R}^n$$

Examples

and from the south pole:



$$\sigma_S : S^n - \{S\} \longrightarrow \mathbb{R}^n$$

$$\sigma_S(x_1, \ldots, x_{n+1}) = \frac{1}{1 + x_{n+1}}(x_1, \ldots, x_n)$$

Examples

- Inverse stereographic projections:

$$\sigma_N^{-1}(x_1, \ldots, x_n) = \frac{1}{\left(\sum_{i=1}^n x_i^2\right) + 1}\left(2x_1, \ldots, 2x_n, \left(\sum_{i=1}^n x_i^2\right) - 1\right)$$

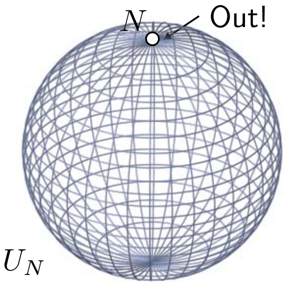
and

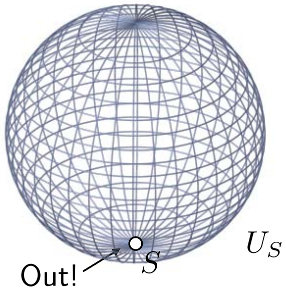
$$\sigma_S^{-1}(x_1, \ldots, x_n) = \frac{1}{\left(\sum_{i=1}^n x_i^2\right) + 1}\left(2x_1, \ldots, 2x_n, -\left(\sum_{i=1}^n x_i^2\right) + 1\right).$$

Examples

- Consider the open cover consisting of

$$U_N = S^n - \{N\} \qquad \text{and} \qquad U_S = S^n - \{S\}\,.$$

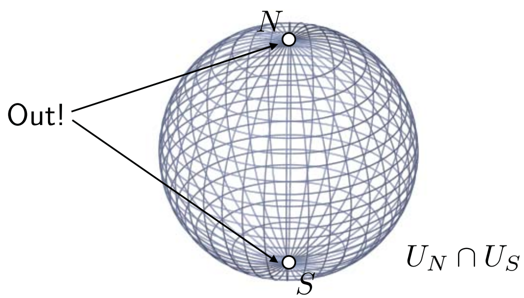




Examples

- On the overlap,

$$U_N \cap U_S = S^n - \{N, S\}\,.$$



Examples

- The transition maps

$$\sigma_S \circ \sigma_N^{-1} = \sigma_N \circ \sigma_S^{-1}$$

are given by

$$(x_1, \ldots, x_n) \mapsto \frac{1}{\sum_{i=1}^{n} x_i^2}(x_1, \ldots, x_n)\,.$$

Examples

- Consequently,

$$(U_N, \sigma_N) \quad \text{and} \quad (U_S, \sigma_S)$$

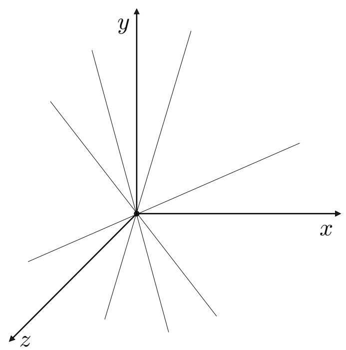
form a smooth atlas for S^n .

- So, the sphere is a smooth manifold.

Examples

- The real projective space, $\mathbb{RP}^n$.
- This is the space of all lines through the origin of $\mathbb{R}^{n+1}$.

$\mathbb{RP}^2$



Examples

- Equivalent definition:

Define an equivalence relation on nonzero vector in $\mathbb{R}^{n+1}$ as follows:

$$u \sim v \quad \text{iff} \quad v = \lambda u, \text{ for some } \lambda \neq 0 \in \mathbb{R}\,.$$

- Denote the equivalence class of $(x_1, \ldots, x_{n+1})$ by

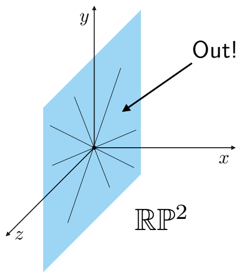
$$(x_1 : \cdots : x_{n+1})$$

also called homogeneous coordinates.

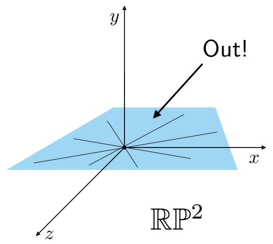
Examples

- Let

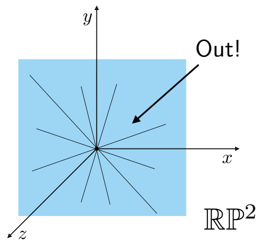
$$U_i = \{(x_1 : \cdots : x_{n+1}) \in \mathbb{RP}^n \mid x_i \neq 0\} \,.$$



$U_1 = \{(x : y : z) \in \mathbb{RP}^2 \mid x \neq 0\}$



$U_2 = \{(x : y : z) \in \mathbb{RP}^2 \mid y \neq 0\}$

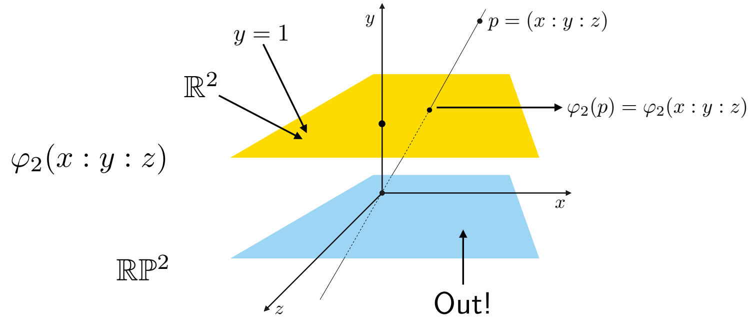


$U_3 = \{(x : y : z) \in \mathbb{RP}^2 \mid z \neq 0\}$

Examples

- Define $\varphi_i : U_i \to \mathbb{R}^n$ by

$$\varphi_i(x_1 : \cdots : x_{n+1}) = \left(\frac{x_1}{x_i}, \ldots, \frac{x_{i-1}}{x_i}, \frac{x_{i+1}}{x_i}, \ldots, \frac{x_{n+1}}{x_i} \right) .$$



Examples

- The inverse maps are given by

$$\psi_i(x_1, \ldots, x_n) = (x_1 : \cdots : x_{i-1} : 1 : x_{i+1} : \cdots : x_n) \,.$$

- On the overlap, $U_i \cap U_j$,

$$(\varphi_j \circ \varphi_i^{-1})(x_1, \ldots, x_n) = \left(\frac{x_1}{x_j}, \ldots, \frac{x_{i-1}}{x_j}, \frac{1}{x_j}, \frac{x_i}{x_j}, \ldots, \frac{x_{j-1}}{x_j}, \frac{x_{j+1}}{x_j}, \ldots, \frac{x_n}{x_j} \right)$$

- As these maps are smooth, real projective space is a smooth manifold.

Conclusions

- In the next part of the tutorial, we will show that a manifold can be **reconstructed** from its transition functions.

- Such a construction was first proposed by Andre Weil around 1944 in his book, *Foundations of Algebraic Geometry*.

- A similar approach was used to construct fiber bundles in the 1950's (Steenrod).

Constructing Manifolds from Sets of Gluing Data

Jean Gallier
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Outline

Motivations

- Recall that **we want to define a surface** S that approximates the underlying surface, $|S_T|$, of a given polygonal surface (mesh), S_T.
- More specifically, we want to build a C^k two-dimensional manifold in $\mathbb{R}^3$.
- Our plan is to define S constructively by **building a manifold**.

Motivations

A LITTLE PROBLEM:

Our definition of manifold is not constructive: it states what a manifold is by assuming it already exists! So, for our purposes, it is not useful!

THE KEY IDEA:

The notion of a **set of gluing data**.

Sets of Gluing Data

Let n and k be integers such that $n \geq 1$ and $k \geq 1$ (or $k = \infty$).

A set of gluing data is a triple

$$\mathcal{G} = \big((\Omega_i)_{i \in I}, (\Omega_{ij})_{(i,j) \in I \times I}, (\varphi_{ji})_{(i,j) \in K \times K}\big)$$

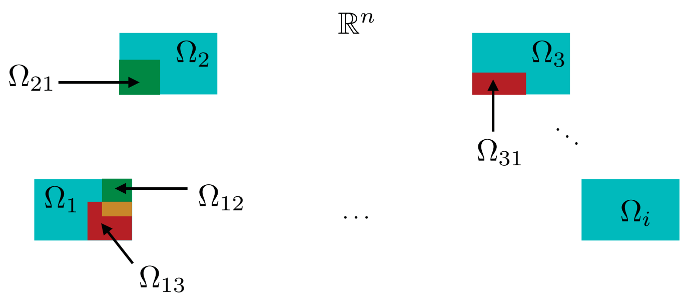
satisfying the following properties, where I and K are countable sets and I is non-empty:

Sets of Gluing Data

(1) For every $i \in I$, the set Ω_i is a non-empty open subset of $\mathbb{R}^n$ called **parametrization domain**, for short, p-**domain**, and the Ω_i are pairwise disjoint (i.e., $\Omega_i \cap \Omega_j = \emptyset$ for all $i \neq j$).

Sets of Gluing Data

(2) For every pair $(i, j) \in I \times I$, the set Ω_{ij} is an open subset of Ω_i. Furthermore, $\Omega_{ii} = \Omega_i$, and $\Omega_{ji} \neq \emptyset$ if and only if $\Omega_{ij} \neq \emptyset$. Each non-empty Ω_{ij} (with $i \neq j$) is called **gluing domain**.



Sets of Gluing Data

(3) If we let

$$K = \{(i, j) \in I \times I \mid \Omega_{ij} \neq \emptyset\},$$

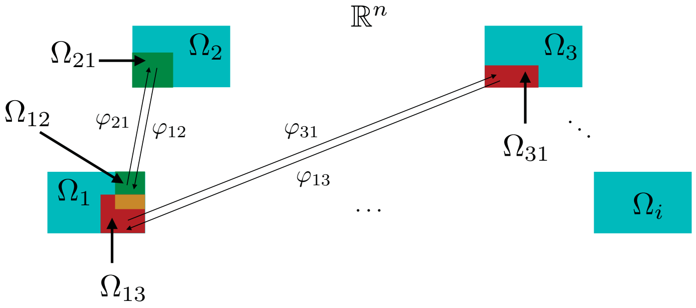
then

$$\varphi_{ji} : \Omega_{ij} \longrightarrow \Omega_{ji}$$

is a C^k bijection for every $(i, j) \in K$, called a **transition function** or **gluing function**.

Transition Functions

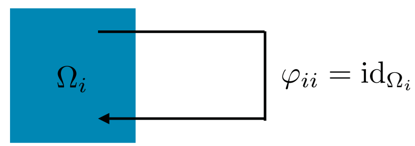
- The transition functions tell us how to glue the p-domains.



Transition Functions

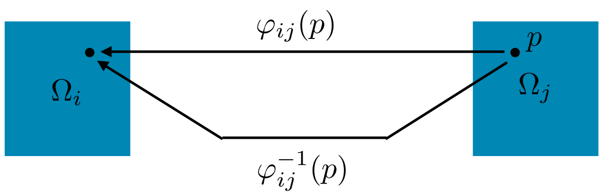
The transition functions must satisfy the following conditions:

(a) $\varphi_{ii} = \mathrm{id}_{\Omega_i}$, for all $i \in I$,



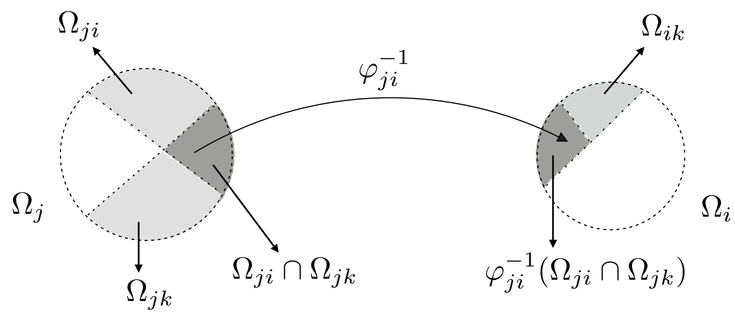
Transition Functions

(b) $\varphi_{ij} = \varphi_{ji}^{-1}$, for all $(i,j) \in K$, and



Transition Functions

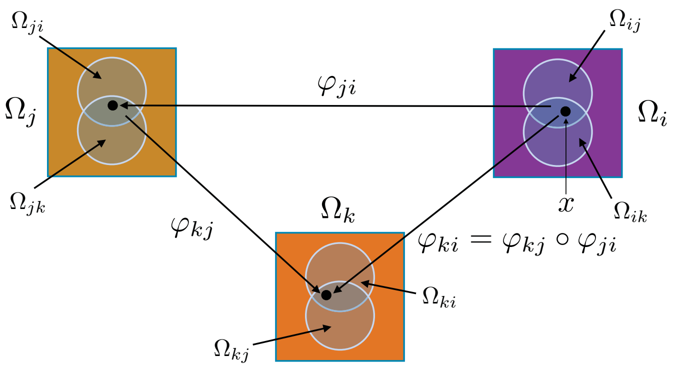
(c) for all i, j, and k, if $\Omega_{ji} \cap \Omega_{jk} \neq \emptyset$ then $\varphi_{ji}^{-1}(\Omega_{ji} \cap \Omega_{jk}) \subseteq \Omega_{ik}$ and $\varphi_{ki}(x) = \varphi_{kj} \circ \varphi_{ji}(x)$, for all $x \in \varphi_{ji}^{-1}(\Omega_{ji} \cap \Omega_{jk})$.



The Cocycle Condition

The "evil" cocycle condition

$\varphi_{ki}(x) = \varphi_{kj} \circ \varphi_{ji}(x)$, for all $x \in \varphi_{ji}^{-1}(\Omega_{ji} \cap \Omega_{jk})$.

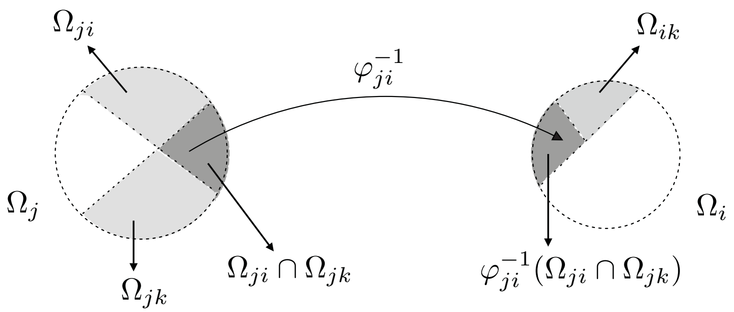


The Cocycle Condition

- The cocycle condition implies conditions (a) and (b).
- Previous versions found in the literature are often incorrect.

The Cocycle Condition

- This is because the transition maps are only partial functions!



Parametric Pseudo-Manifolds

- The question now becomes:

Given a set of gluing data, $\mathcal{G}$, can we build a manifold from it?

- Indeed, such a manifold is built by a **quotient construction.**
- We form the disjoint union of the Ω_i and we identify Ω_{ij} with Ω_{ji} using φ_{ji}, an equivalence relation, $\sim$. We form the quotient

$$M_{\mathcal{G}} = \left(\coprod_i \Omega_i \right) / \sim, .$$

Parametric Pseudo-Manifolds

Theorem 1 [Gallier, Siqueira, and Xu, 2008]

For every set of gluing data,

$$\mathcal{G} = \left((\Omega_i)_{i\in I}, (\Omega_{ij})_{(i,j)\in I\times I}, (\varphi_{ji})_{(i,j)\in K\times K}\right) ,$$

there is an n-dimensional C^k manifold, $M_{\mathcal{G}}$, whose transition functions are the φ_{ji}'s.

Parametric Pseudo-Manifolds

REMARK:

A condition on the gluing data is needed to make sure that $M_{\mathcal{G}}$ is Hausdorff. Since it is quite technical, we will not show it here.

Parametric Pseudo-Manifolds

Theorem 1 is very nice, but ...

- Our proof is not constructive;

- $M_{\mathcal{G}}$ is an abstract entity, which may not even be compact, orientable, etc.

So, the question that remains is **how** to build a *concrete* manifold.

Let us first formalize our notion of "concreteness".

Parametric Pseudo-Manifolds

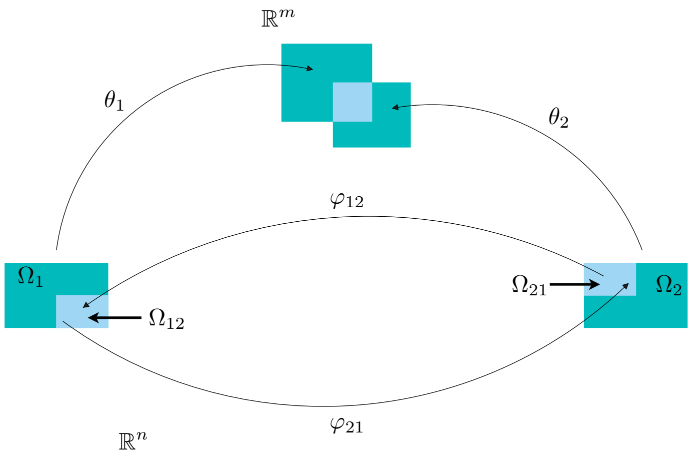
Let n, m, and k be integers, with $m > n \geq 1$ and $k \geq 1$ or $k = \infty$.

A **parametric C^k pseudo-manifold of dimension n in $\mathbb{R}^m$** is a pair,

$$\mathcal{M} = (\mathcal{G}, (\theta_i)_{i\in I}) ,$$

such that $\mathcal{G} = \left((\Omega_i)_{i\in I}, (\Omega_{ij})_{(i,j)\in I\times I}, (\varphi_{ij})_{(i,j)\in K\times K}\right)$ is a set of gluing data, for some finite I, and each θ_i is a C^k function, $\theta_i : \Omega_i \to \mathbb{R}^m$, called a **parametrization** such that the following holds:

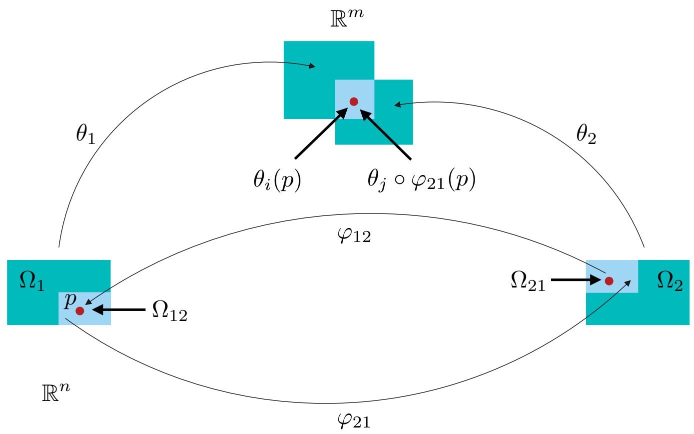
Parametric Pseudo-Manifolds



- When $m = 3$ and $n = 2$, we say that $\mathcal{M}$ is a **parametric pseudo-surface**.

Parametric Pseudo-Manifolds

(C) For all $(i, j) \in K$, we have $\theta_i = \theta_j \circ \varphi_{ji}$.



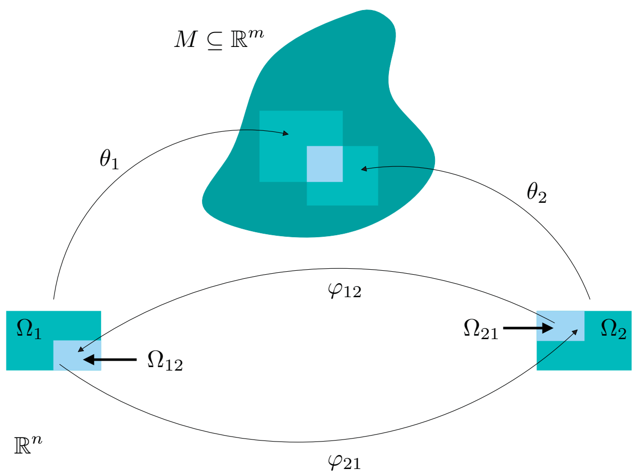
Parametric Pseudo-Manifolds

- The subset

$$M = \bigcup_{i \in I} \theta_i(\Omega_i)$$

of $\mathbb{R}^m$ is called the **image** of the parametric pseudo-manifold.

Parametric Pseudo-Manifolds



Parametric Pseudo-Manifolds

REMARK:

There is a (unique) surjective map:

$$\Theta : M_{\mathcal{G}} \longrightarrow M\,.$$

Parametric Pseudo-Manifolds

We proved that M can be given a manifold structure if we require the θ_i's to be bijective and to satisfy the following conditions:

(C') For all $(i, j) \in K$,

$$\theta_i(\Omega_i) \cap \theta_j(\Omega_j) = \theta_i(\Omega_{ij}) = \theta_j(\Omega_{ji})\,.$$

(C") For all $(i, j) \notin K$,

$$\theta_i(\Omega_i) \cap \theta_j(\Omega_j) = \emptyset\,.$$

Conclusions

- We can *build* a parametric pseudo-manifold (PPM) from a set of gluing data and, *under certain conditions*, the image of a PPM can be given the structure of a manifold.
- In the following lecture, we describe a new constructive approach to define a set of gluing data from a triangle mesh.
- We also describe how to build a parametric C^∞ pseudo-surface from the set of gluing data. The image of this parametric pseudo-surface approximates the vertices of the mesh.

Fitting Surfaces to Polygonal Meshes (Part I)

Marcelo Siqueira
UFMS

Outline

- The Surface Fitting Problem
- Building a Set of Gluing Data

The Surface Fitting Problem

Given a mesh S_T in $\mathbb{R}^3$, a positive integer k, and a positive real number ϵ, our goal here is to fit a C^k surface, S, in $\mathbb{R}^3$ to S_T.

The Manifold-Based Approach:

We solve the fitting problem by defining a C^k parametric pseudo-surface, $\mathcal{M}$, such that S is the image, M, of $\mathcal{M}$ in $\mathbb{R}^3$.

The Surface Fitting Problem

Key Idea:

- Use S_T to define the set of gluing data, $\mathcal{G}$, of $\mathcal{M}$.

  ↑ **TOPOLOGY**

- Use $|S_T|$ to define the set of parametrizations, $(\theta_i)_{i \in I}$, of $\mathcal{M}$.

  ↑ **GEOMETRY**

Building a Set of Gluing Data

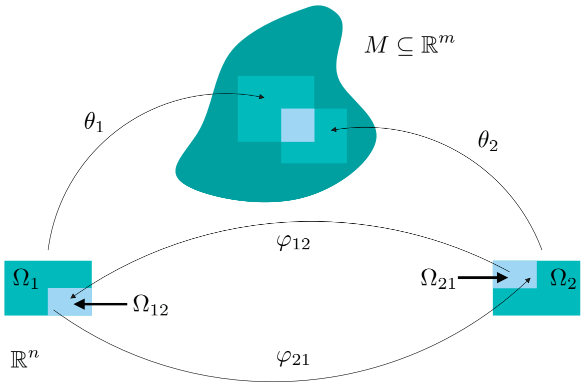
To define $\mathcal{G}$, we must

- define the p-domains, $(\Omega_i)_{i \in I}$,
- define the gluing domains, $(\Omega_{ij})_{(i,j) \in I \times I}$,
- define the transition functions, $(\varphi_{i,j})_{(i,j) \in K \times K}$.

$$\mathcal{G} = \big((\Omega)_{i \in I}, (\Omega_{i,j})_{(i,j) \in I \times I}, (\varphi_{i,j})_{(i,j) \in K \times K} \big)$$

Building a Set of Gluing Data

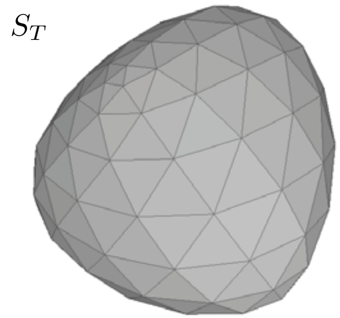
The BIG PICTURE



Building a Set of Gluing Data

p-**Domains**

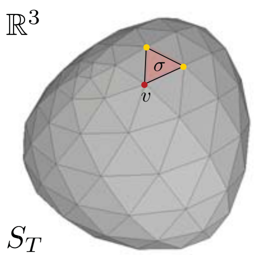
Assume that S_T is a **triangle** mesh (i.e., a simplicial surface).



Building a Set of Gluing Data

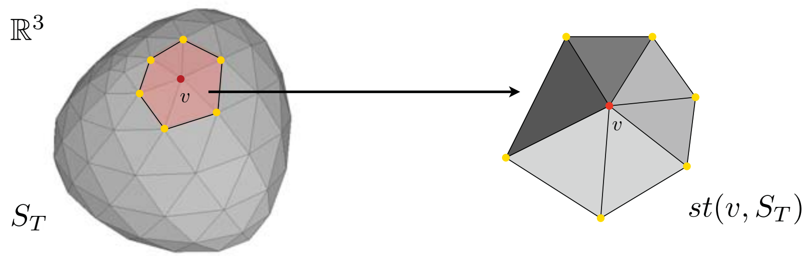
Let

$$I = \{(\sigma, v) \mid \sigma \text{ is a triangle of } S_T \text{ and } v \text{ is a vertex of } \sigma\}.$$



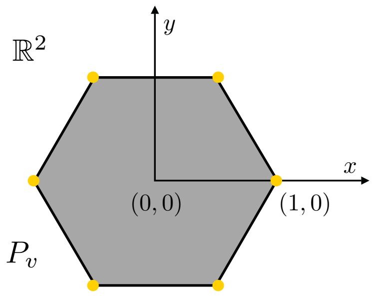
Building a Set of Gluing Data

For every vertex, v, of S_T, consider its **star**, $st(v, S_T)$:



Building a Set of Gluing Data

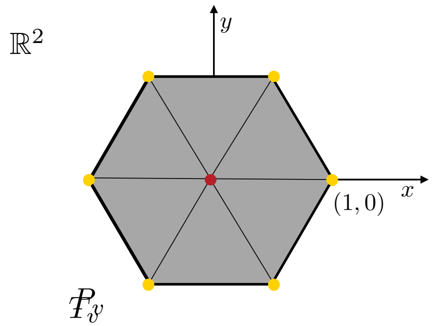
Define the **P-polygon, P_v, associated with** v as the m_v-gon inscribed in the circle of radius 1 and centered at the origin in $\mathbb{R}^2$:



m_v is the **degree** of v in S_T.

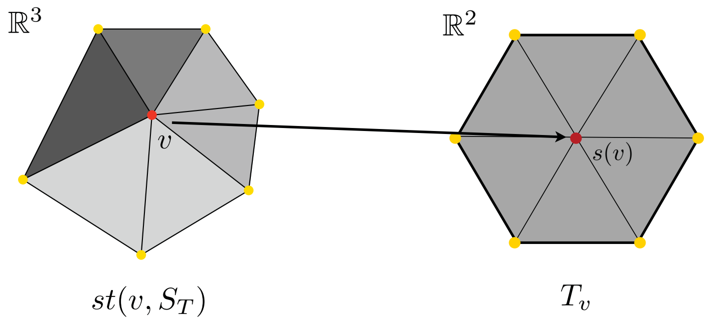
Building a Set of Gluing Data

Define the **triangulation, T_v, associated with** v by adding straight edges (diagonals) connecting the barycenter of P_v to its vertices:



Building a Set of Gluing Data

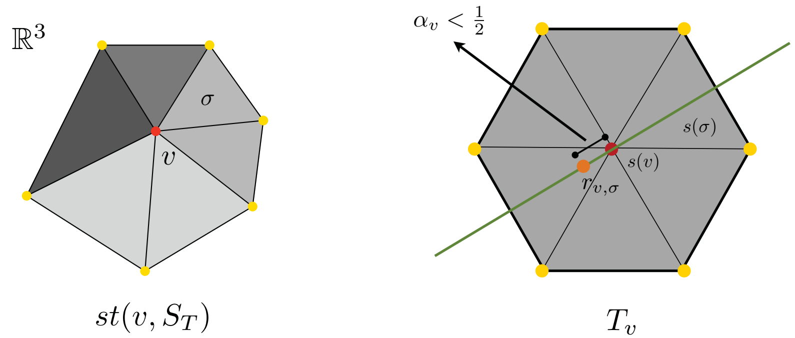
Remark: T_v is a parametrization of $st(v, S_T)$ in $\mathbb{R}^2$:



$s : st(v, S_T) \to T_v$

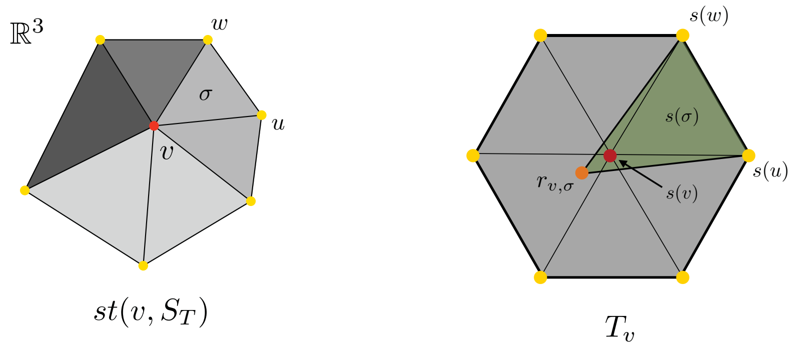
Building a Set of Gluing Data

For each triangle σ of S_T and vertex v of σ, we define the **overlapping point**, $r_{v,\sigma}$, associated with $s(\sigma)$ in T_v, as follows:



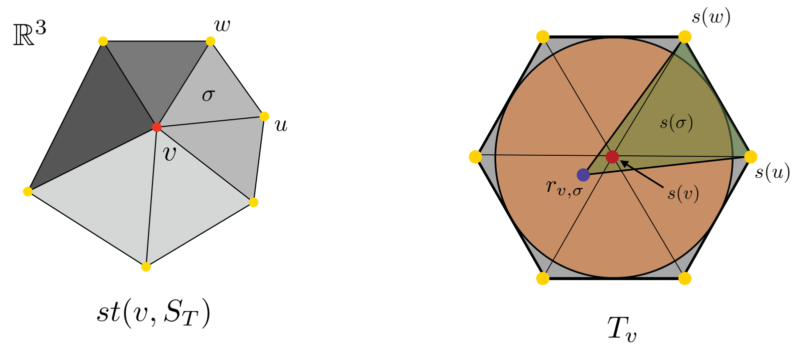
Building a Set of Gluing Data

If $\sigma = [v, u, w]$ then consider the triangle $[r_{\sigma,v}, s(u), s(w)]$:



Building a Set of Gluing Data

Consider the circle, C_v, inscribed in P_v:

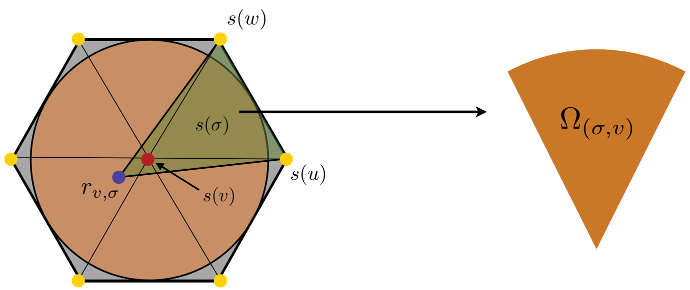


Building a Set of Gluing Data

We let $\Omega_{(\sigma,v)}$ be

$$C_v \cap \text{int}([r_{v,\sigma}, s(u), s(w)]) \,,$$

where $\text{int}([r_{v,\sigma}, s(u), s(w)])$ is the interior of $[r_{v,\sigma}, s(u), s(w)]$.



Building a Set of Gluing Data

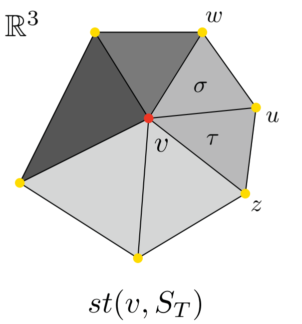
Remark:

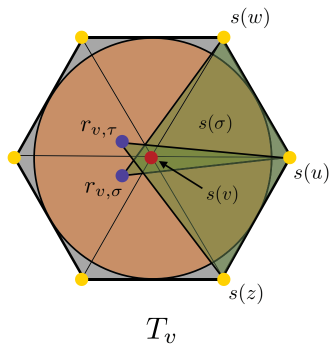
From Jean Gallier's lecture, we should have

$$\Omega_{(\sigma,v)} \cap \Omega_{(\tau,u)} = \emptyset \,,$$

for any two pairs, (σ, v) and (τ, u), in I. Did I make it right?

Building a Set of Gluing Data





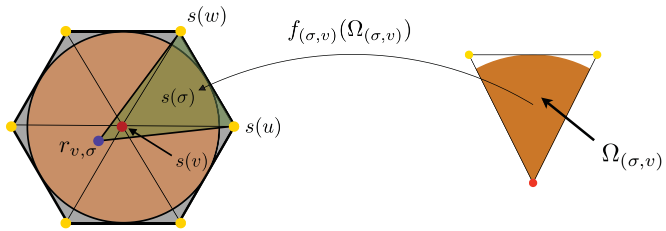
Clearly, $\Omega_{(\sigma,v)} \cap \Omega_{(\tau,v)} \neq \emptyset$.

Building a Set of Gluing Data

So, I did NOT make it right.

What now?

We can *fix* that by letting $\Omega_{(\sigma,v)}$ be a set inside a triangle which is the inverse image of $[r_{v,\sigma}, s(u), s(w)]$ under a rigid transformation!

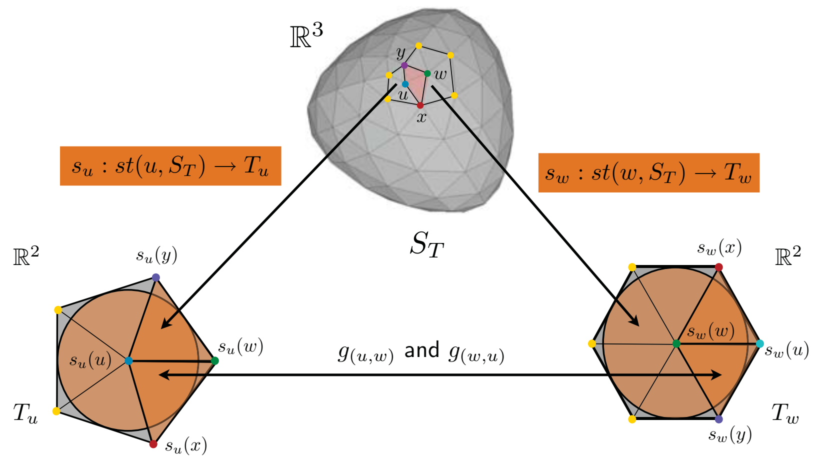


Building a Set of Gluing Data

Since I is a finite set and the "enclosing" triangles are compact, we can certainly separate each p-domain from the others in $\mathbb{R}^2$.

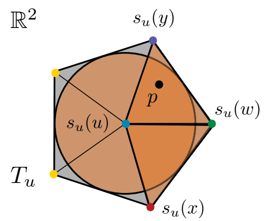
Building a Set of Gluing Data

Gluing domains



Building a Set of Gluing Data

Let p be a point in the region $C_u \cap [s_u(u), s_u(x), s_u(w), s_u(y)]$.



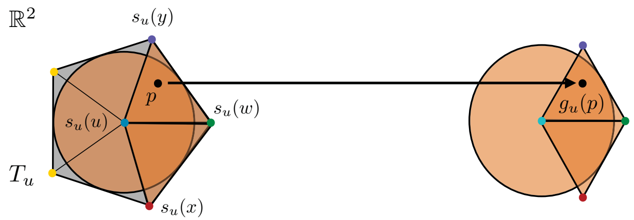
Let (θ, r) be the polar coordinates of point p with respect to the local coordinate system of P_u (i.e., origin at $s_u(u) = (0,0)$).

Building a Set of Gluing Data

Let $g_u : [0, 2\pi) \times \mathbb{R}_+ \to [0, 2\pi) \times \mathbb{R}_+$ be the map

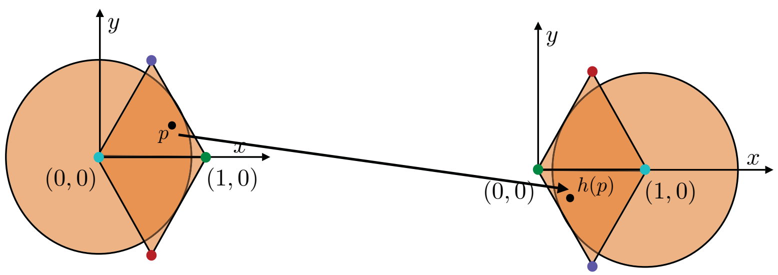
$$g_u(p) = g_u((\theta, r)) = \left(\frac{6}{m_u} \cdot \theta \, , \, \frac{\cos(\frac{\pi}{6})}{\cos(\frac{\pi}{m_u})} \cdot r \right) ,$$

where m_u is the degree of u.



Building a Set of Gluing Data

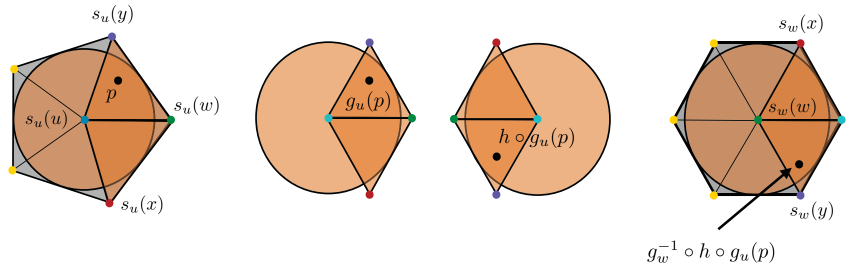
Let $h : \mathbb{R}^2 \to \mathbb{R}^2$ be the map $h(p) = h((x, y)) = (1 - x, -y)$:



Building a Set of Gluing Data

Finally, we define $g_{(u,w)} : [0, 2\pi) \times \mathbb{R}_+ \to [0, 2\pi) \times \mathbb{R}_+$ as

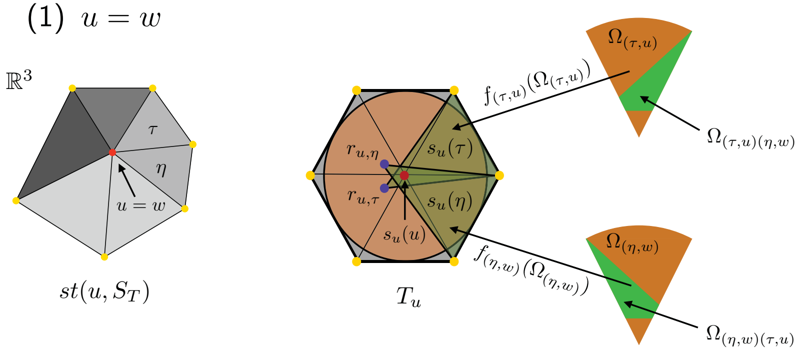
$$g_{(u,w)}(p) = g_{(u,w)}((\theta, r)) = g_w^{-1} \circ h \circ g_u((\theta, r)) \, .$$



Building a Set of Gluing Data

For any two $(\tau, u), (\eta, w) \in I$, we define $\Omega_{(\tau,u)(\eta,w)}$ as follows:

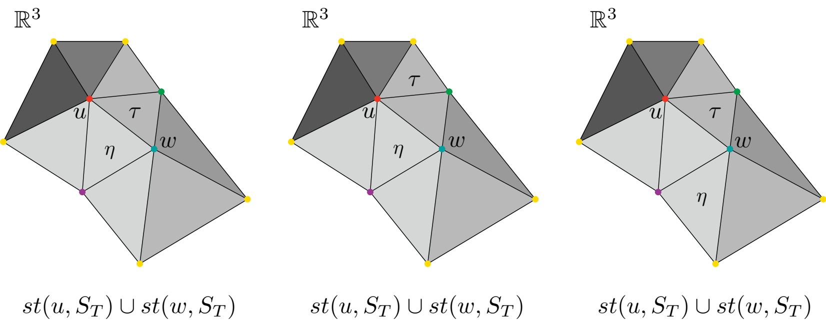
(1) $u = w$



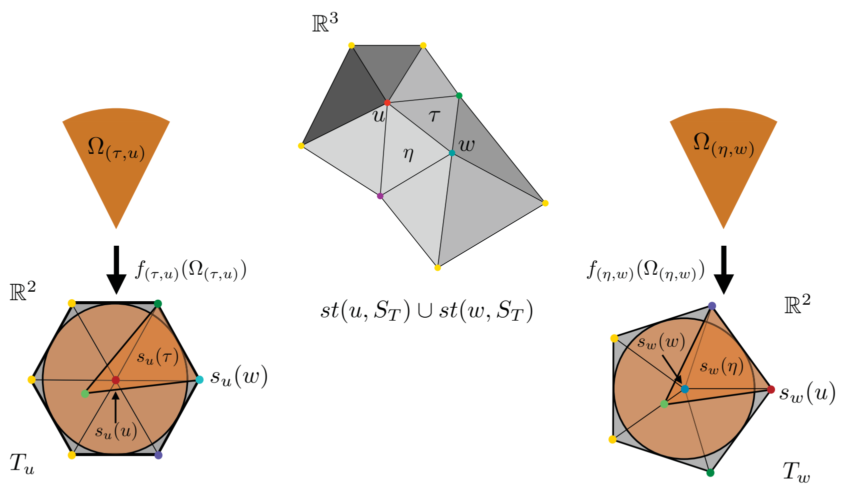
$$\Omega_{(\tau,u)(\eta,w)} = f^{-1}_{(\tau,u)}(f_{(\tau,u)}(\Omega_{\tau,u}) \cap f_{(\eta,w)}(\Omega_{\eta,w}))$$

Building a Set of Gluing Data

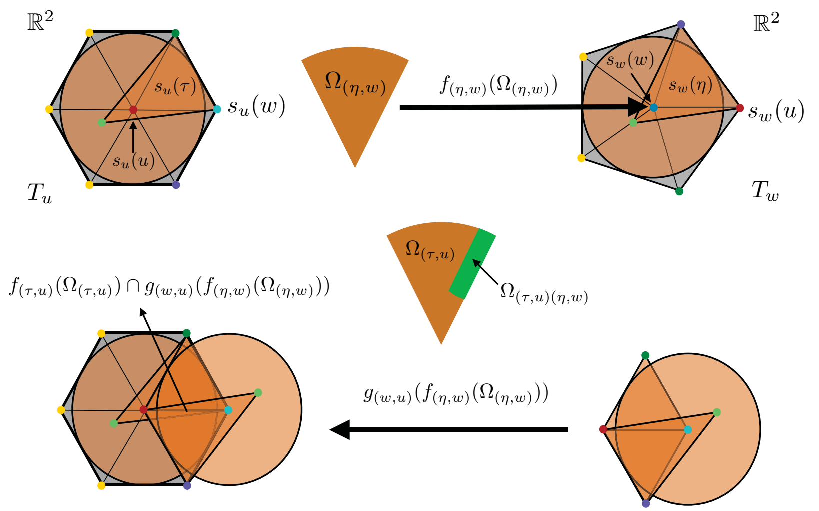
(2) $u \neq w$ and w is a vertex of τ or u is a vertex of η



Building a Set of Gluing Data

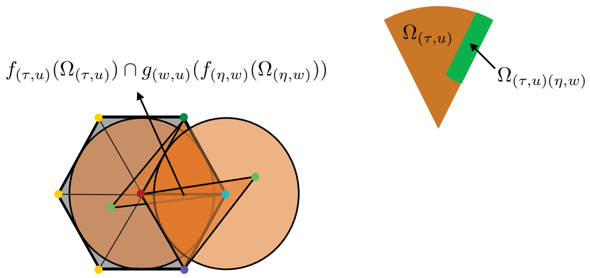


Building a Set of Gluing Data



Building a Set of Gluing Data

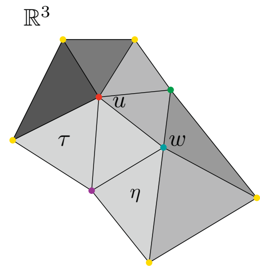
$$\Omega_{(\tau,u)(\eta,w)} = f^{-1}_{(\tau,u)}(f_{(\tau,u)}(\Omega_{(\tau,u)}) \cap g_{(w,u)}(f_{(\eta,w)}(\Omega_{(\eta,w)})))$$



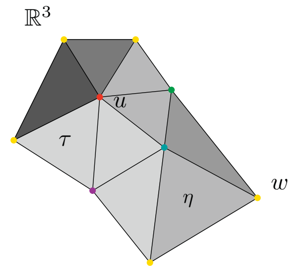
Building a Set of Gluing Data

(3) $u \neq w$ and w is not a vertex of τ nor u is a vertex of η

$$\Omega_{(\tau,u)(\eta,w)} = \emptyset$$



$st(u, S_T) \cup st(w, S_T)$



$st(u, S_T) \cup st(w, S_T)$

Building a Set of Gluing Data

We can show that the above definition of gluing domain satisfies condition (2) of the definition of sets of gluing data we saw before:

(2) For every pair $(i,j) \in I \times I$, the set Ω_{ij} is an open subset of Ω_i. Furthermore, $\Omega_{ii} = \Omega_i$ and $\Omega_{ji} \neq \emptyset$ if and only if $\Omega_{ij} \neq \emptyset$.

Fitting Surfaces to Polygonal Meshes (Part II)

Marcelo Siqueira
UFMS

Outline

- Building a Set of Gluing Data
- The User's Perspective
- Building Parametrizations
- Results
- Conclusions

Building a Set of Gluing Data

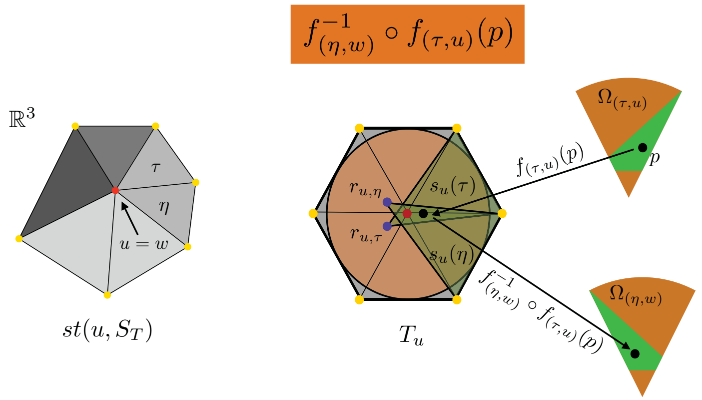
Transition functions

Let

$$K = \{((\tau, u), (\eta, w)) \in I \times I \mid \Omega_{(\tau,u),(\eta,w)} \neq \emptyset\}.$$

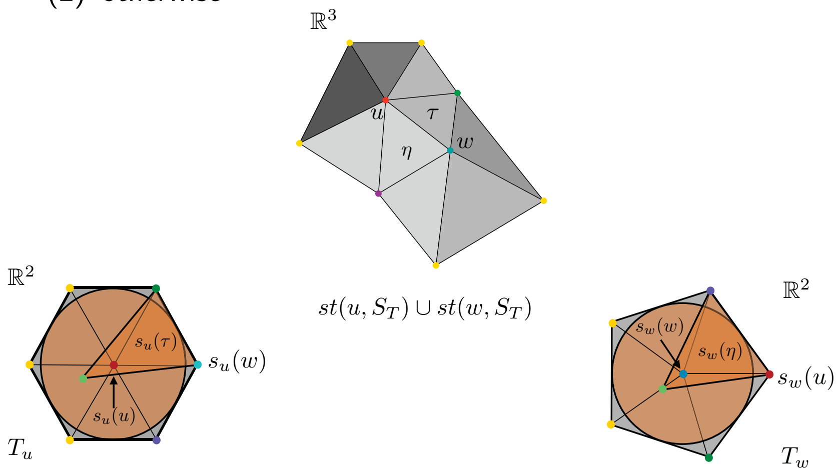
Building a Set of Gluing Data

(1) $u = w$

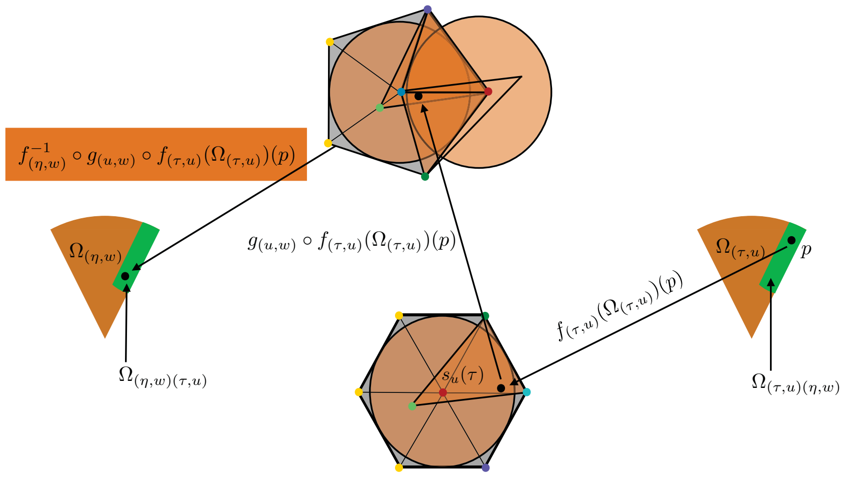


Building a Set of Gluing Data

(2) otherwise



Building a Set of Gluing Data



Building a Set of Gluing Data

For every $((\tau,u),(\eta,w)) \in K$, we define

$$\varphi_{(\eta,w)(\tau,u)} : \Omega_{(\tau,u),(\eta,w)} \rightarrow \varphi_{(\eta,w)(\tau,u)} \,,$$

the transition function from $\Omega_{(\tau,u)}$ to $\Omega_{(\eta,w)}$, to be

$$\varphi_{(\eta,w)(\tau,u)}(p) = \begin{cases} f^{-1}_{(\eta,w)} \circ f_{(\tau,u)}(p) & \text{if } u = w \\ f^{-1}_{(\eta,w)} \circ g_{(u,w)} \circ f_{(\tau,u)}(p) & \text{otherwise}\,, \end{cases}$$

for every $p \in \Omega_{(\tau,u)(\eta,w)}$.

Building a Set of Gluing Data

We can show that the above definition of transition functions satisfies conditions (3)(a)-(c) of the definition of sets of gluing data:

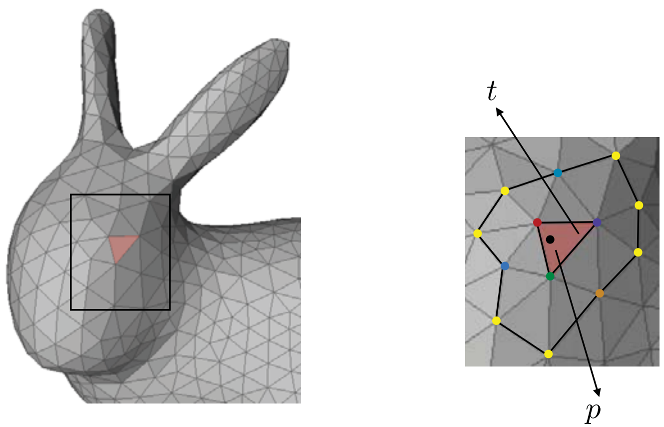
(a) $\varphi_{ii} = \mathrm{id}_{\Omega_i}$, for all $i \in I$,

(b) $\varphi_{ij} = \varphi_{ji}^{-1}$, for all $(i,j) \in K$, and

(c) for all i, j, and k, if $\Omega_{ji} \cap \Omega_{jk} \neq \emptyset$ then $\varphi_{ji}^{-1}(\Omega_{ji} \cap \Omega_{jk}) \subseteq \Omega_{ik}$ and $\varphi_{ki}(x) = \varphi_{kj} \circ \varphi_{ji}(x)$, for all $x \in \varphi_{ji}^{-1}(\Omega_{ji} \cap \Omega_{jk})$.

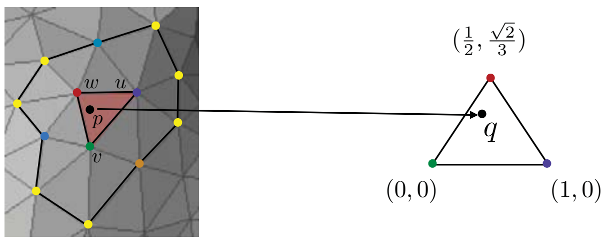
User's Perspective

Let t be a triangle in S_T and p be any point in t:



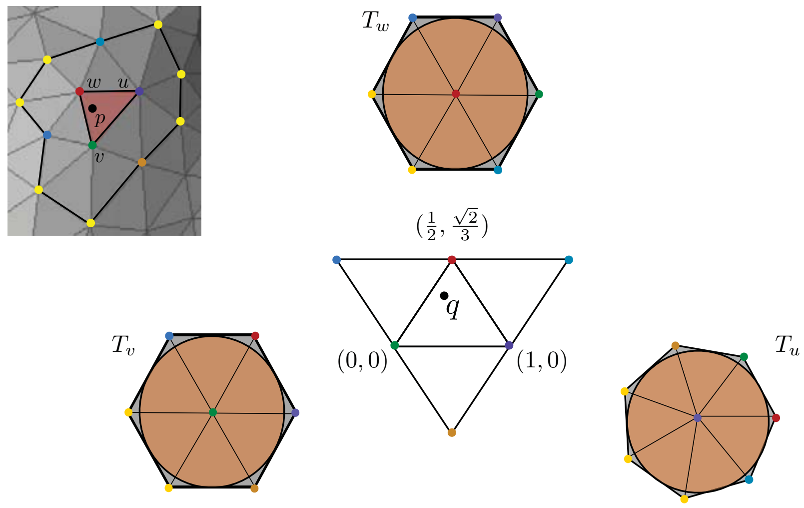
User's Perspective

Map p to an equilateral triangle in $\mathbb{R}^2$.

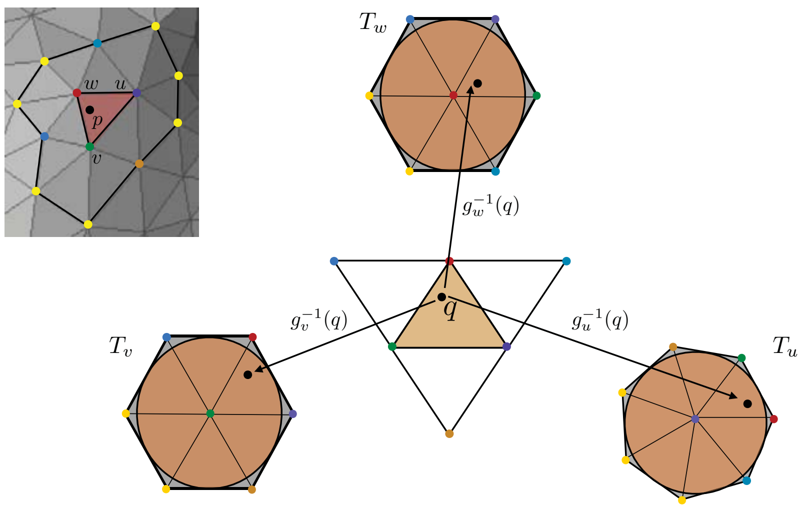


We can do that by using barycentric coordinates.

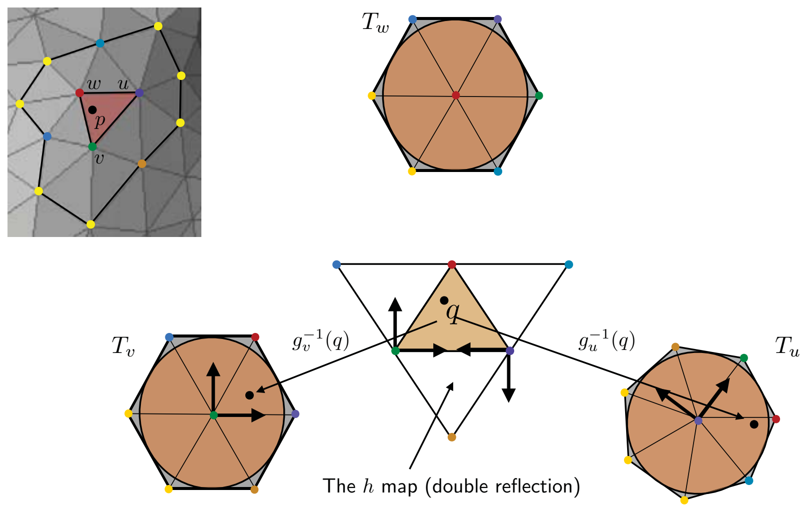
User's Perspective



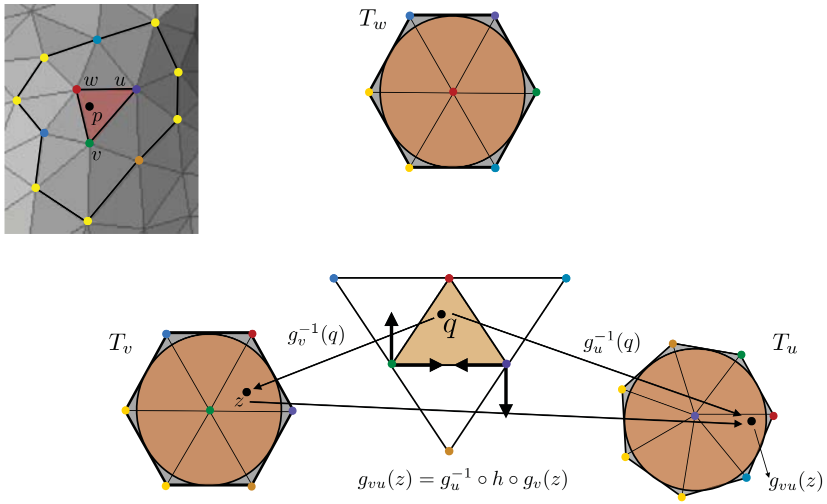
User's Perspective



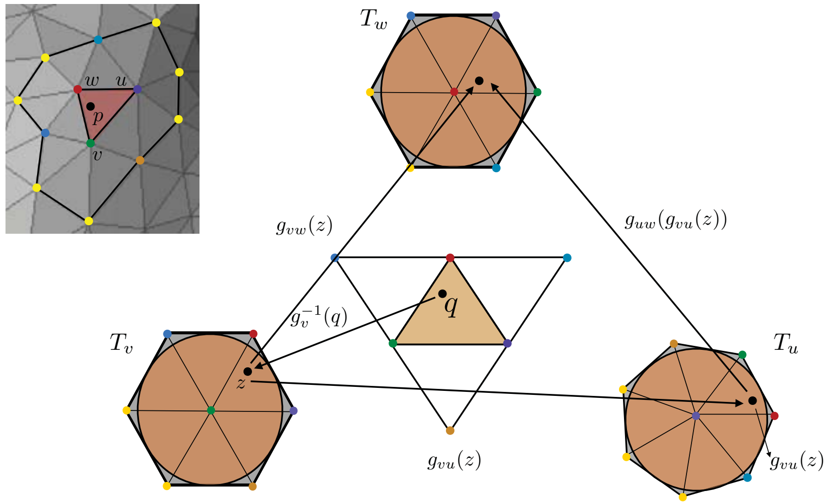
User's Perspective



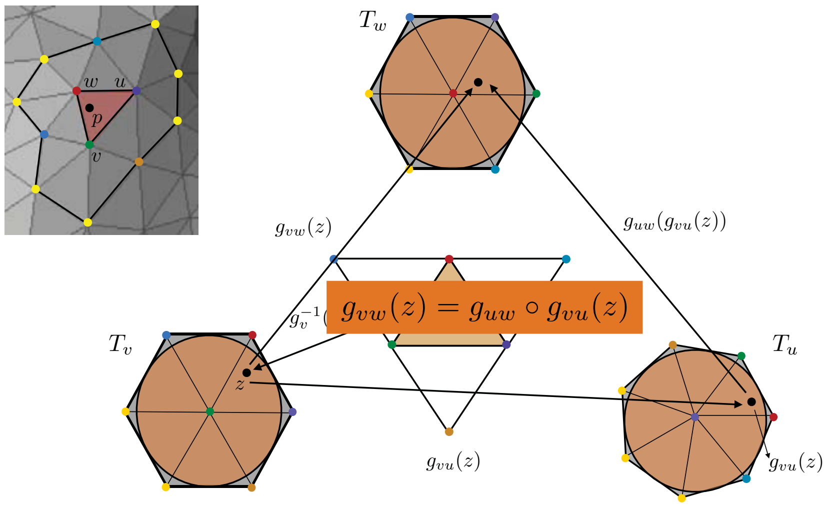
User's Perspective



User's Perspective



User's Perspective

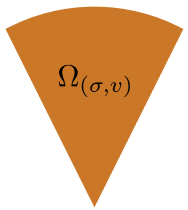


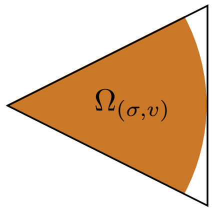
Building Parametrizations

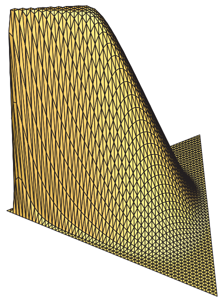
For each $(\sigma, v) \in I$, we define a **weight function**,

$$\gamma_{(\sigma,v)} : \mathbb{R}^2 \to \mathbb{R} \,,$$

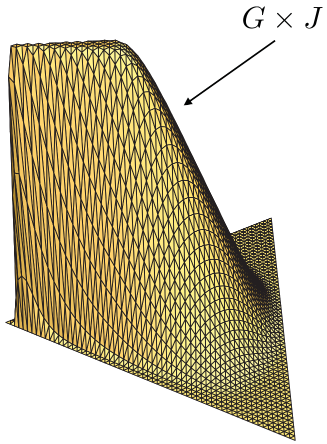
which is the product of two C^∞ curves (and therefore, C^∞ too).

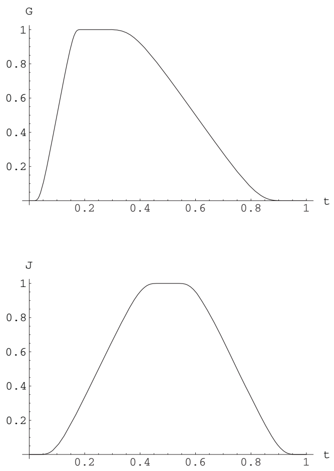






Building Parametrizations



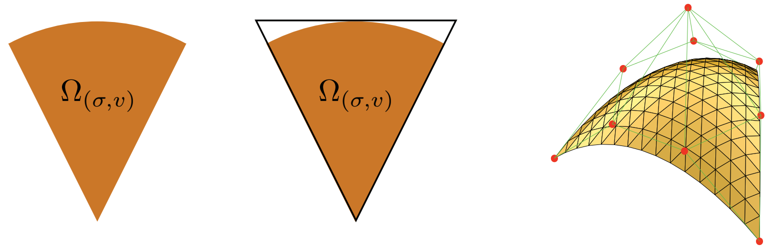


Building Parametrizations

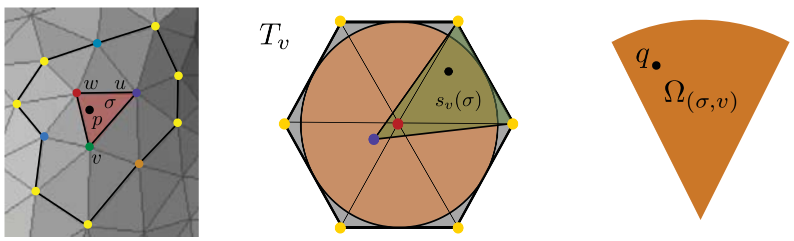
For each $(\sigma, v) \in I$, we define a **Bézier patch**,

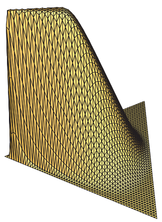
$$\psi_{(\sigma,v)} : \mathbb{R}^2 \longrightarrow \mathbb{R}^3 \,,$$

whose control points are defined in the "envelope" triangle of $\Omega_{(\sigma,v)}$.



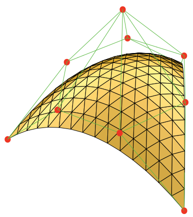
Building Parametrizations



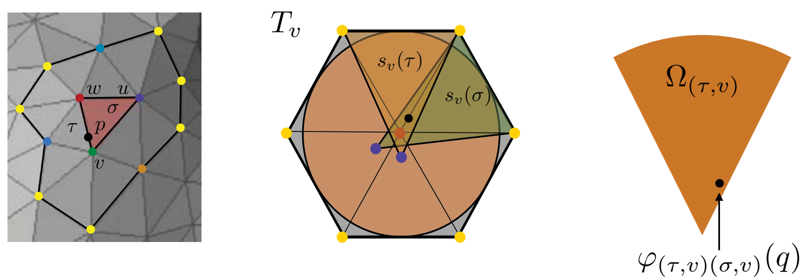


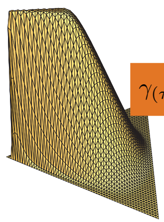
Contribution of $\Omega_{(\sigma,v)}$:

$$\gamma_{(\sigma,v)}(q) \cdot \psi_{(\sigma,v)}(q)$$



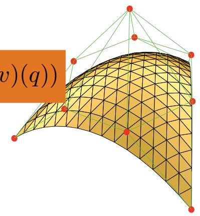
Building Parametrizations



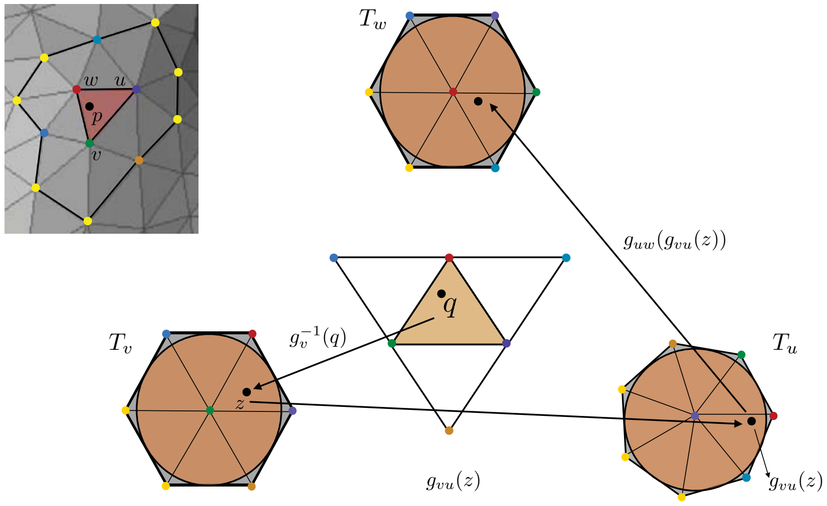


Contribution of $\Omega_{(\tau,v)}$:

$$\gamma_{(\tau,v)}(\varphi(\tau,v)(\sigma,v)(q)) \cdot \psi_{(\tau,v)}(\varphi(\tau,v)(\sigma,v)(q))$$



Building Parametrizations



Building Parametrizations

For each $(\sigma, v) \in I$, we define a **parametrization**,

$$\theta_{(\sigma,v)} : \Omega_{(\sigma,v)} \to \mathbb{R}^3 ,$$

such that for every $p \in \Omega_{(\sigma,v)}$,

$$\theta_{(\sigma,v)}(p) = \sum_{(\tau,u)\in J(p)} \nu_{(\tau,u)}(p) \cdot \psi_{(\tau,u)}(\varphi_{(\tau,u)(\sigma,v)}(p)) ,$$

where

Building Parametrizations

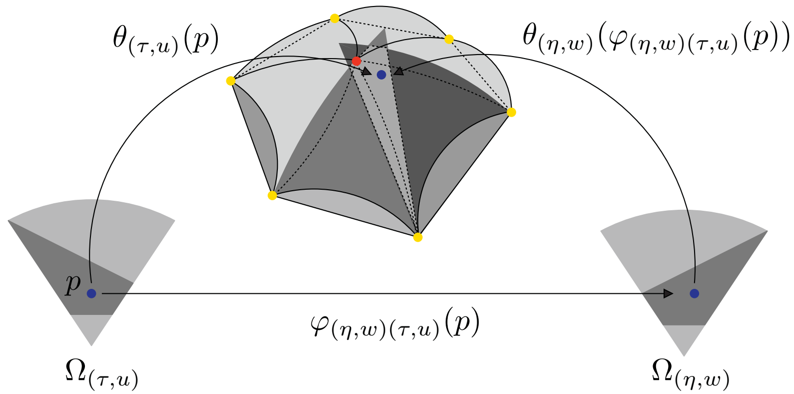
$$\nu_{(\tau,u)}(p) = \frac{\gamma_{(\tau,u)}(\varphi_{(\tau,u)(\sigma,v)}(p))}{\displaystyle\sum_{(\eta,w)\in J(p)} \gamma_{(\eta,w)}(\varphi_{(\eta,w)(\sigma,v)}(p))}$$

and

$$J(p) = \{(\eta, w) \in I \mid p \in \Omega_{(\sigma,v)(\eta,w)}\} .$$

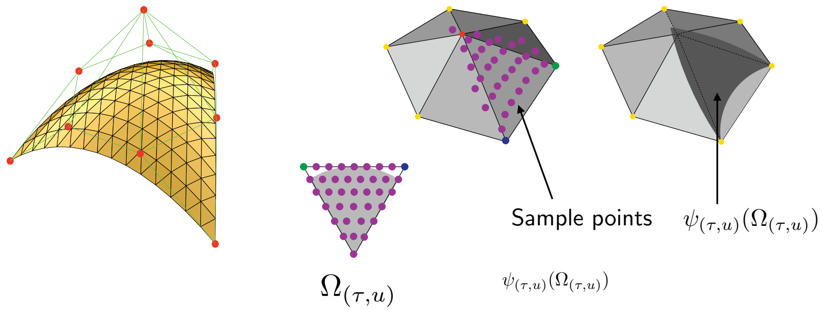
Building Parametrizations

Parametrizations are consistent!



Building Parametrizations

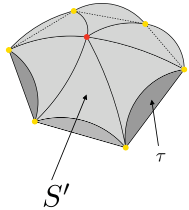
The control points of $\psi_{(\tau,u)}$ are the solutions of a least squares problem.



Building Parametrizations

How can we find the sample points to start with?

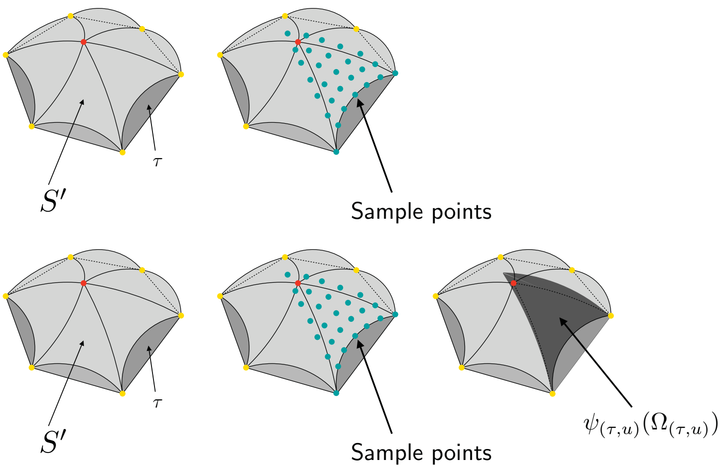
Fit a "curved" surface, S', to S_T and then sample it!



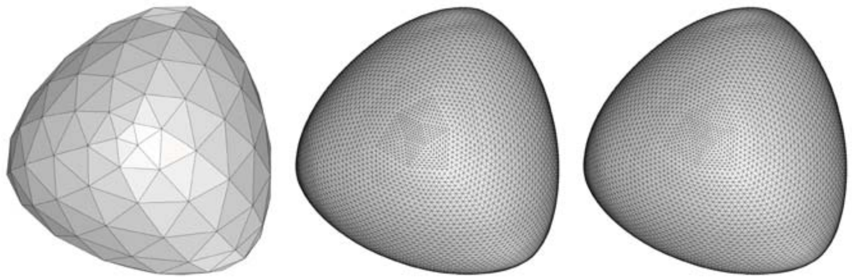
Good choices:

- PN triangle surfaces
- Subdivision surfaces

Building Parametrizations

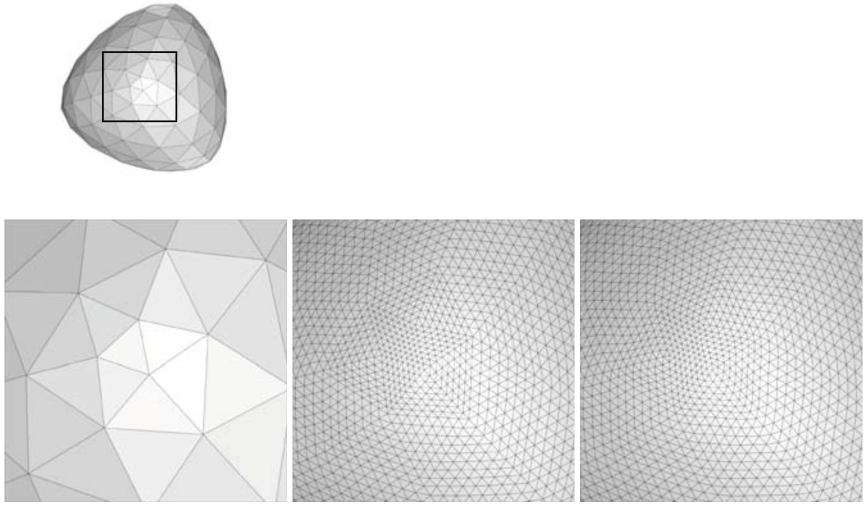


Results



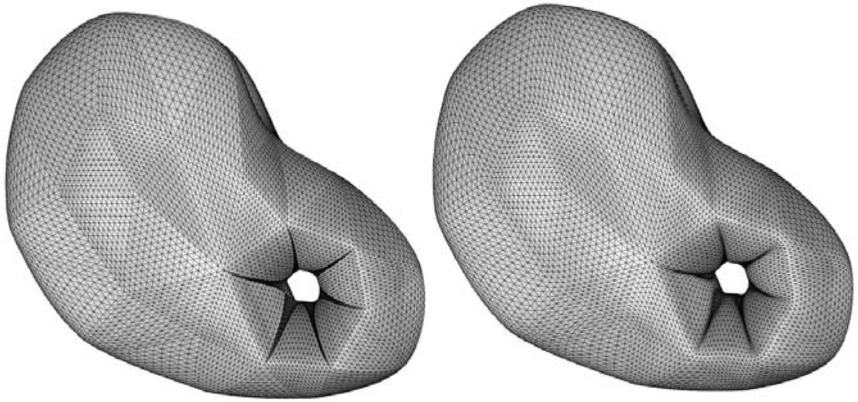
Mesh | PN triangle | PPS

Results

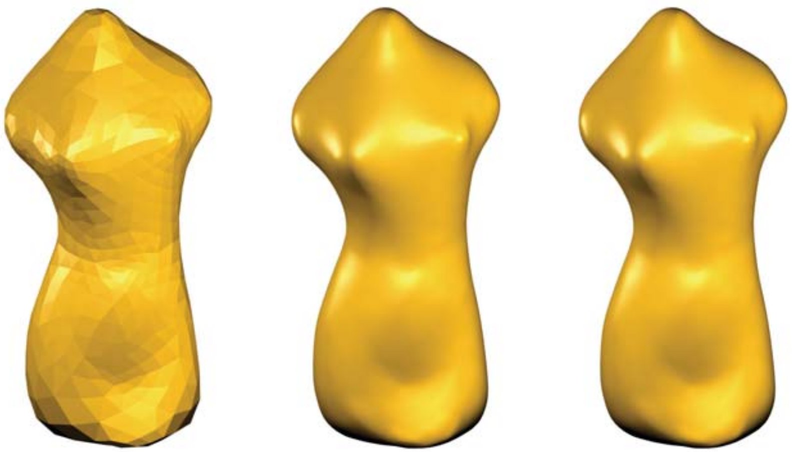


Results

PN triangle PPS

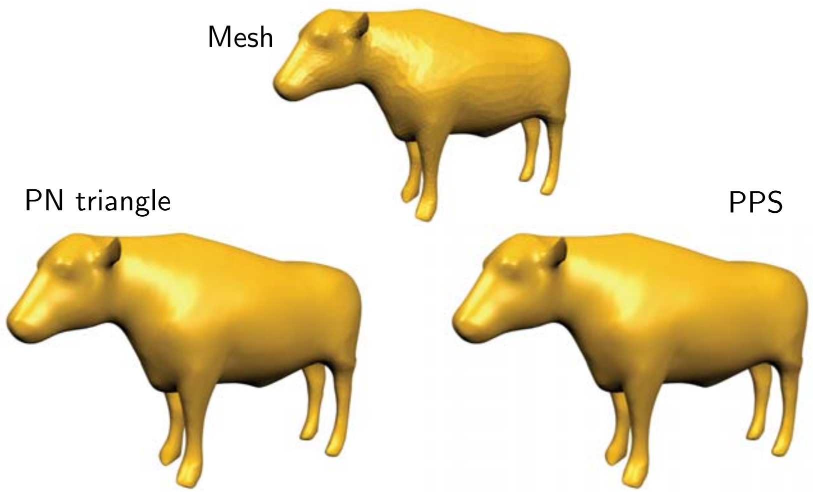


Results



Mesh PN triangle PPS

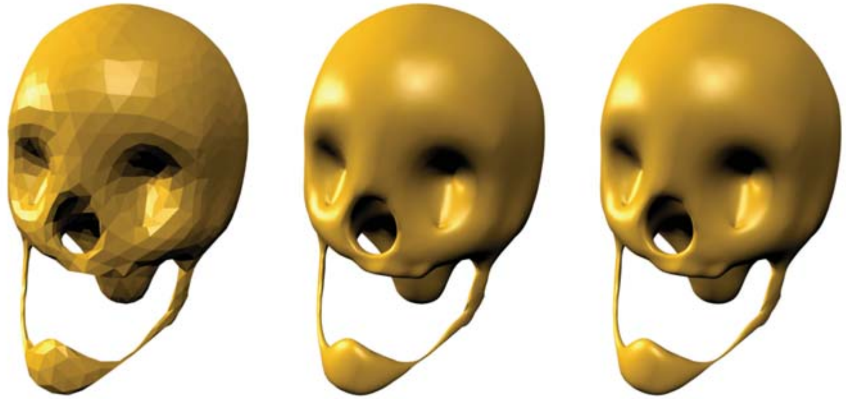
Results



Results

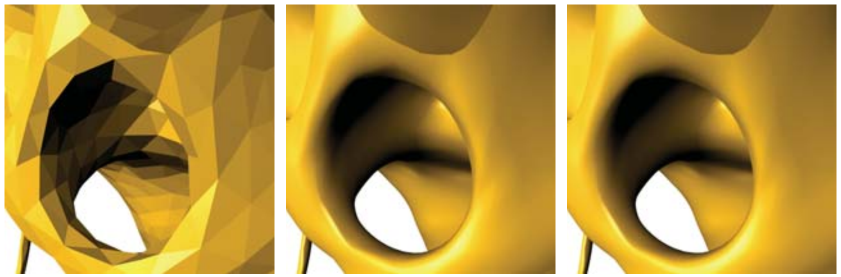
Mesh PN triangle PPS

Results



Mesh PN triangle PPS

Results



Mesh PN triangle PPS

Conclusions

The image of our C^k parametric pseudo-surface is given by

$$M = \bigcup_{(\sigma,v)} \theta_{(\sigma,v)}(\Omega_{(\sigma,v)})\,.$$

The map $\theta_{(\sigma,v)}$ is actually C^∞.

There are $3 \times n_t$ p-domains and Bézier patches in our construction, where n_t is the number of triangles of the input mesh, S_T.

Conclusions

Unfortunately, the map $\theta_{(\sigma,v)}$ is NOT polynomial.

OPEN PROBLEM: Can we make it polynomial?

Conclusions

Recall that

$$\theta_{(\sigma,v)}(p) = \sum_{(\tau,u)\in J(p)} \nu_{(\tau,u)}(p) \cdot \psi_{(\tau,u)}(\varphi_{(\sigma,v)(\tau,u)}(p)),$$

where

$$\nu_{(\tau,u)}(p) = \frac{\gamma_{(\tau,u)}(\varphi_{(\tau,u)(\sigma,v)}(p))}{\displaystyle\sum_{(\eta,w)\in J(p)} \gamma_{(\eta,w)}(\varphi_{(\eta,w)(\sigma,v)}(p))}$$

and

$$J(p) = \{(\eta,w) \in I \mid p \in \Omega_{(\sigma,v)(\eta,w)}\}.$$

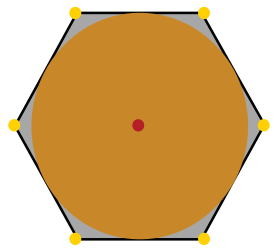
Conclusions

We can easily make $\gamma_{(\tau,u)}$ a C^k rational polynomial, for any finite k.

However, the difficult lies in making $\varphi_{(\tau,u)(\sigma,v)}$ (rational) polynomial!.

Conclusions

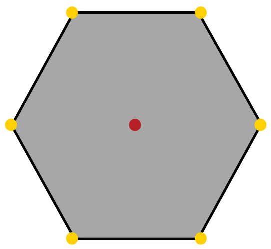
We can create a much simpler construction by letting the p-domains be the inscribed circles of the P-polygons, as shown below:



The transition maps do not change, but the shape functions do!

Conclusions

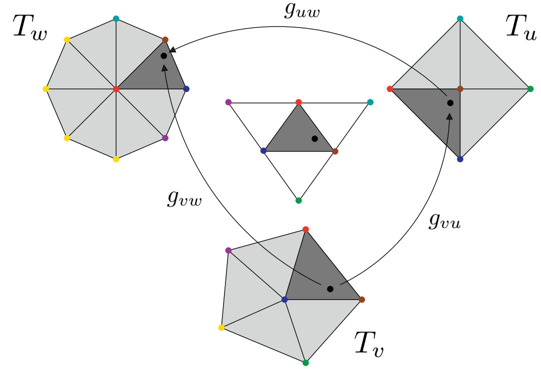
Why didn't we let the interior of the P-polygons be the p-domains?



Simple answer: we failed to figure out the transition maps!

Conclusions

OPEN PROBLEM: Can you find a **simple** C^∞ bijective map g satisfying $g_{vw} = g_{uw} \circ g_{vu}$ (this has to do with the cocycle condition)?



Conclusions

For a good survey on the existing constructions, see

- Cindy M. Grimm and Denis Zorin. Surface Modeling and Parametrization with Manifolds. In ACM SIGGRAPH 2006 Courses (SIGGRAPH'06), pages 1-81, New York, NY, USA, 2006. ACM Press.

Adaptive Manifold Fitting (Part I)

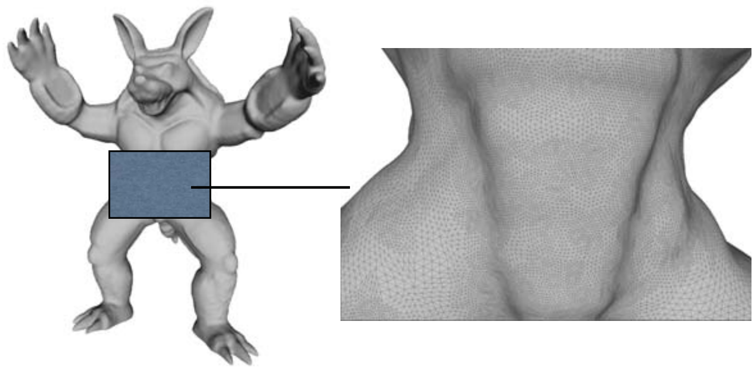
Luiz Velho
IMPA

Outline

- Fitting Surfaces to Very Large Meshes
- Multiresolution Operators
- Building Base Meshes by Simplification
- Adaptive Mesh Refinement
- Conclusions

Surface Fitting

- Very Large Meshes (10^6 vertices)
  - Challenging Problem!



Manifolds and Fitting

- Basic Setting
  - Gluing Data proportional to Mesh Size
- Problem: *Very Large Meshes*
  - Computationally Inefficient
  - Do not Exploit Approximation Power
- Solution:
  - Adaptation

Adaptive Fitting

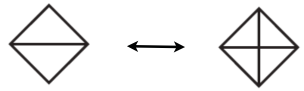
- Optimization Formulation:
  - Given an Approximation Error ϵ
  - Find $\mathcal{M}$ with Smallest Number of Charts
- Strategy:
  - Combine
    - Multiresolution Structure
    - Manifold Surface Approximation

Multiresolution Framework

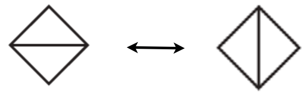
- Simplicial Multi-triangulation
  - Stellar Theory
- Building Base Meshes
  - Surface Simplification
- Adaptive Fitting
  - *4-8* Refinement

Stellar Theory

- Topological Operators
- Edge Split and Weld
  - Change Mesh Resolution



- Edge Flip
  - Change Mesh Connectivity



Stellar Simplification

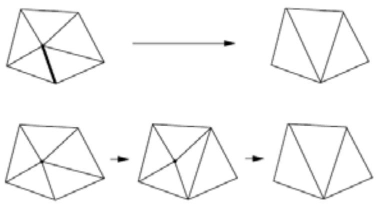
- Basic Elements:

I. Operator Factorization

- Edge Collapse

↕

- Flip + Weld



II. Quadric Error Metric

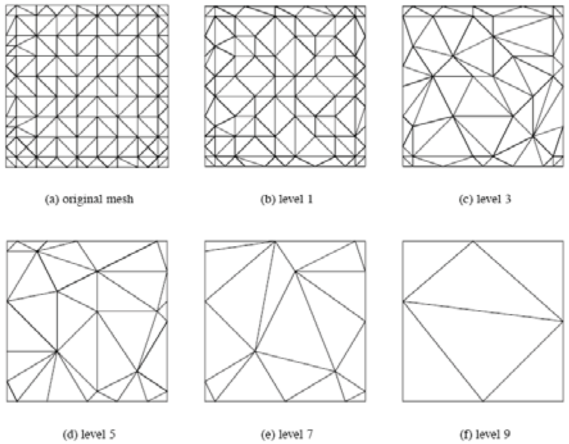
Basic Algorithm

- Repeat for *N* Resolution Levels
  1. Rank Vertices Based on Quadric Error
  2. Select Independent Set of Clusters
  3. Simplify Mesh using Stellar Operators

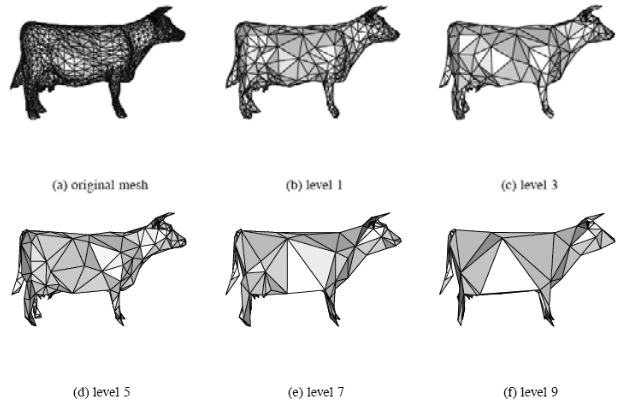
✳ *Properties*

- *Logarithmic Height*
- *Good Aspect Ratios*

Example 1: Plane

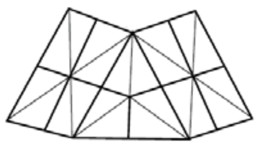


Example 2: Cow

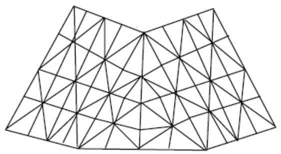


Variable Resolution Mesh

- Underlying Semi-Regular Structure
  - Tri-quad Base Mesh

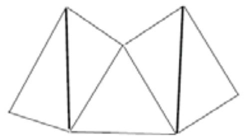


  - *4-8* Subdivision

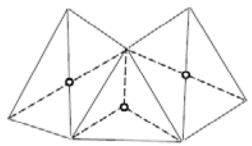


Building the Base Mesh

1. Two-Face Clusters + Single Triangles

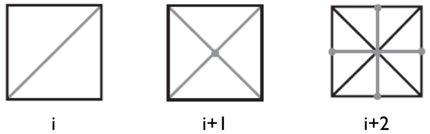


2. Barycenter Subdivision

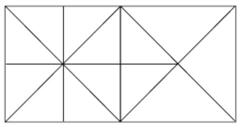


4-8 Subdivision

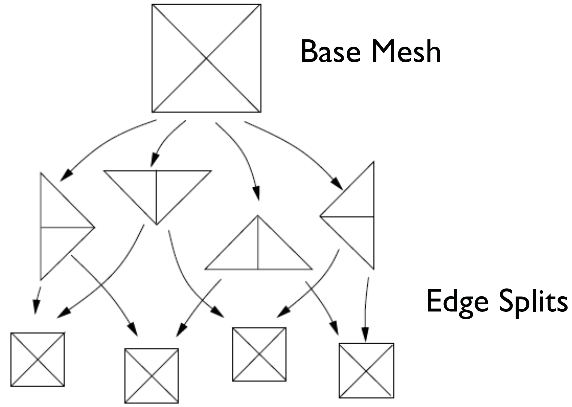
- Interleaved Binary Subdivision



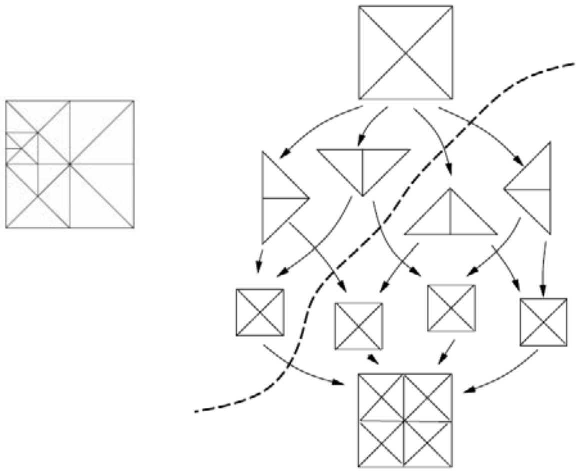
- Non-Uniform Refinement



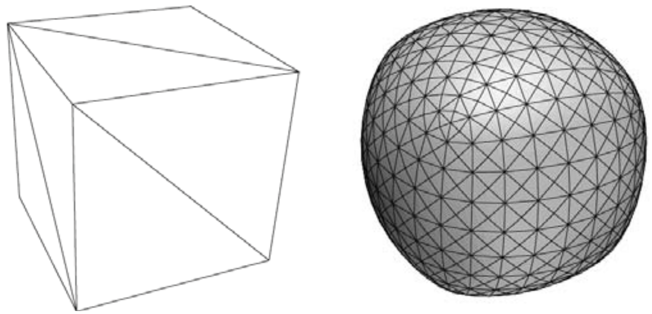
Binary Multi-Triangulation



Adaptive Refinement

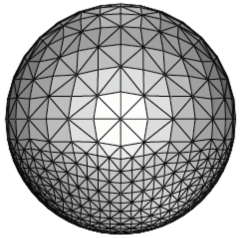


Example I: Uniform

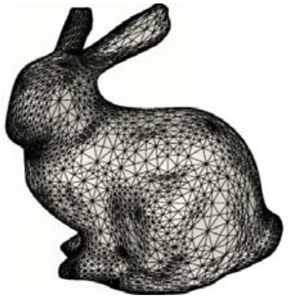


Example 2: Adaptive

- Application-Dependent Criteria



Spatial Selection



Curvature

Conclusions

- Simplicial Multiresolution
  - Powerful Mechanism for Adaptation
- First Part of the Solution for Surface Fitting
  - Simplification
  - Adaptive Refinement
- Second Part (Next)
  - Geodesic Parametrization
  - Bezier Approximation

Adaptive Manifold Fitting (Part II)

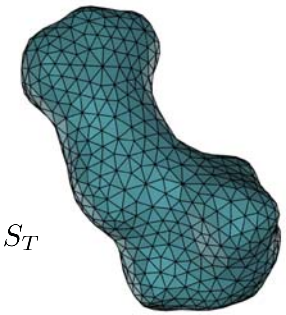
Dimas Martínez Morera
UFAL

Outline

- The Surface Fitting Problem
- Adaptive Fitting
- Discrete Geodesics
- Conclusions

The Surface Fitting Problem

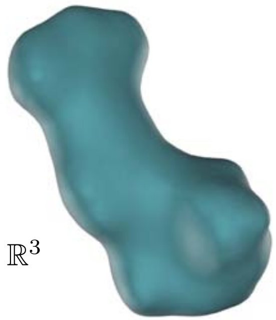
We are a given a piecewise-linear surface, S_T, in $\mathbb{R}^3$, with an empty boundary, a positive integer k, and a positive number ϵ, ...



S_T

The Surface Fitting Problem

We want to find a C^k surface $S \subset \mathbb{R}^3$ ...



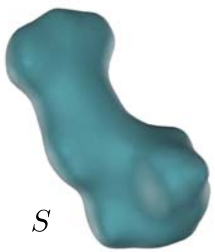
$S \subset \mathbb{R}^3$

The Surface Fitting Problem

such that there exists a homeomorphism, $h : S \to |S_T|$, satisfying

$$\|h(v) - v\| \leq \epsilon \, ,$$

for every vertex v of S_T.

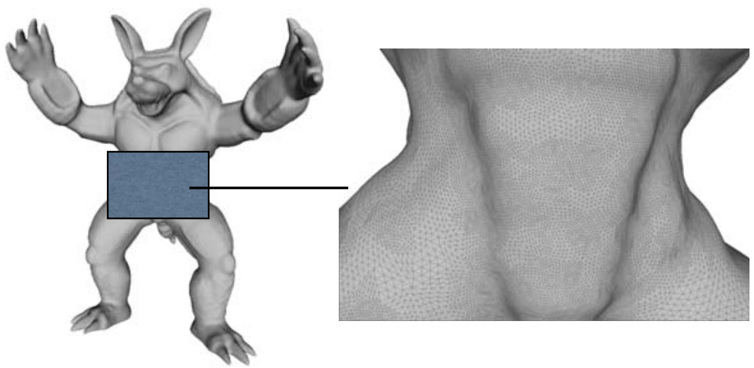


S

The Surface Fitting Problem

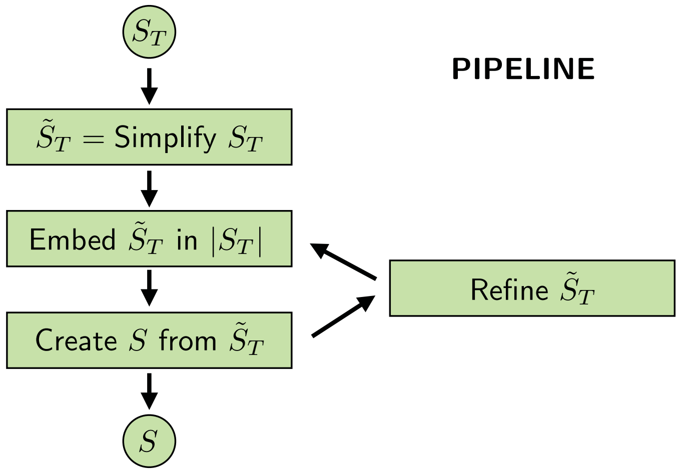
REMARK:

S_T is expected to be "very large" ($\sim 10^6$ vertices).



Adaptive Fitting

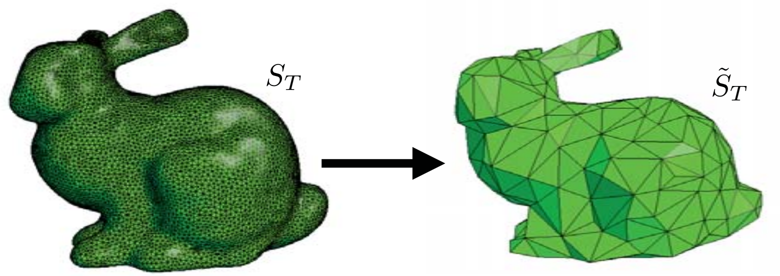
PIPELINE



Adaptive Fitting

S_T → $\tilde{S}_T$ = Simplify S_T

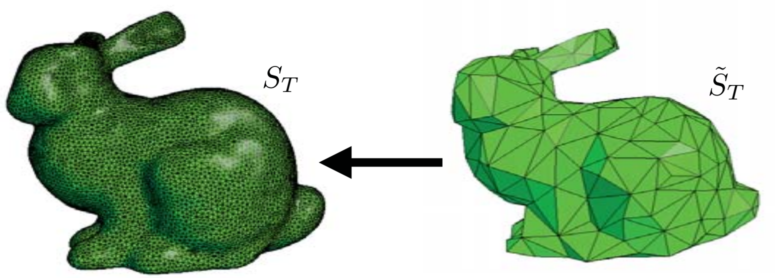
- Four-Face Clusters Algorithm



Adaptive Fitting

Embed $\tilde{S}_T$ in $|S_T|$

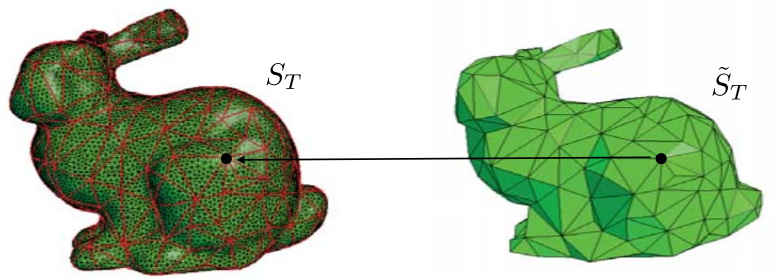
- Each edge of $\tilde{S}_T$ is embedded in $|S_T|$ as a "geodesic".



Adaptive Fitting

REMARK:

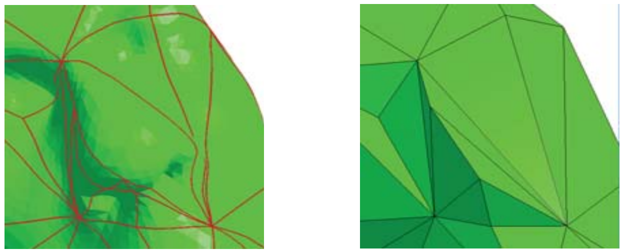
The vertices of $\tilde{S}_T$ ARE vertices of S_T.



Adaptive Fitting

PROBLEM:

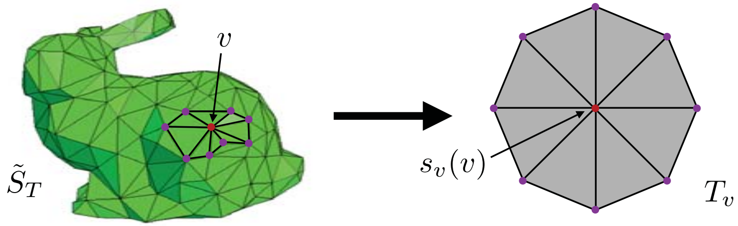
When defining geodesic triangles, we can violate the manifold property of the geodesic mesh, as illustrated by the figure below:



Adaptive Fitting

Create S from $\tilde{S}_T$

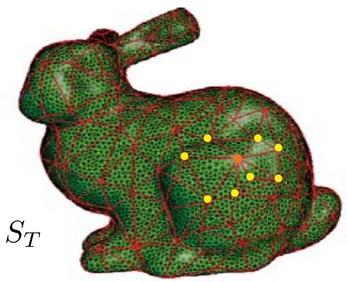
- For each vertex v of $\tilde{S}_T$, we consider the P-polygon, P_v, of v in $\mathbb{R}^2$, and the standard triangulation, T_v, of the P-polygon P_v.



Adaptive Fitting

Create S from $\tilde{S}_T$

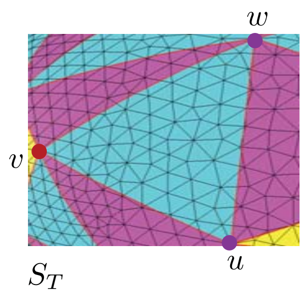
- Consider the embedding of the star, $st(v, \tilde{S}_T)$, of v in S_T.

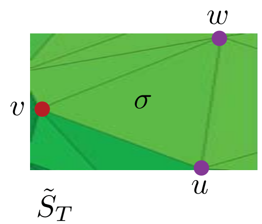


Adaptive Fitting

Create S from $\tilde{S}_T$

- Map the vertices of S_T bounded by the embedding of $st(v, \tilde{S}_T)$ to T_v.

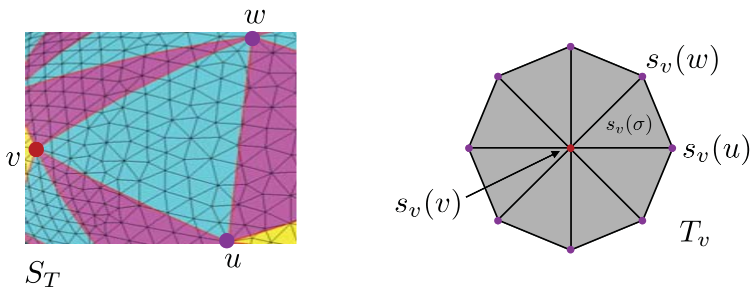




Adaptive Fitting

Create S from $\tilde{S}_T$

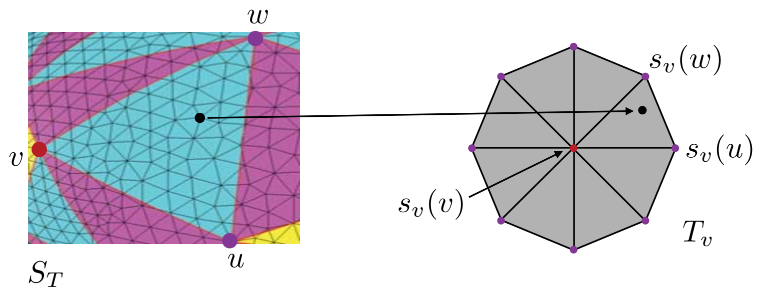
- Map the vertices of S_T bounded by the embedding of $st(v, \tilde{S}_T)$ to T_v.



Adaptive Fitting

Create S from $\tilde{S}_T$

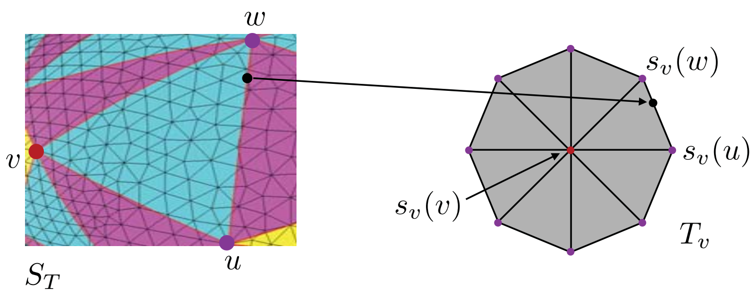
- Map the vertices of S_T bounded by the embedding of $st(v, \tilde{S}_T)$ to T_v.



Adaptive Fitting

Create S from $\tilde{S}_T$

- Points where geodesics intersect edges of S_T are also mapped to T_v.



Adaptive Fitting

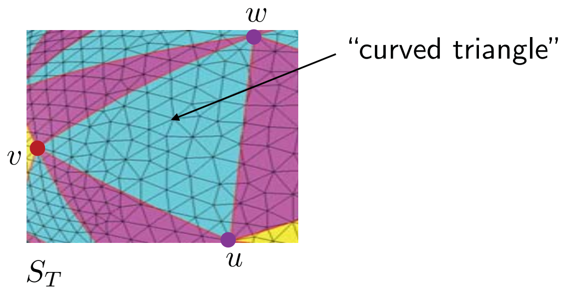
Create S from $\tilde{S}_T$

- How is this mapping done?

Adaptive Fitting

Create S from $\tilde{S}_T$

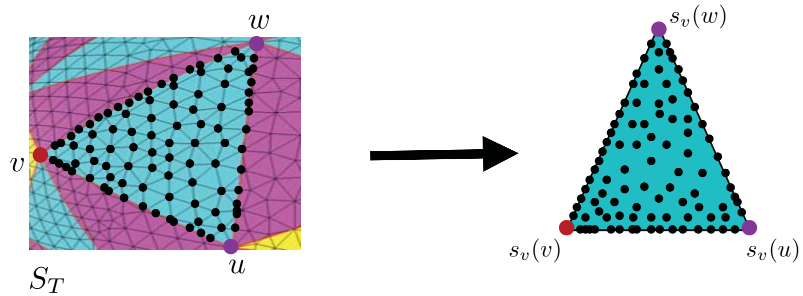
- We map the vertices in each "curved" triangle separately.



Adaptive Fitting

Create S from $\tilde{S}_T$

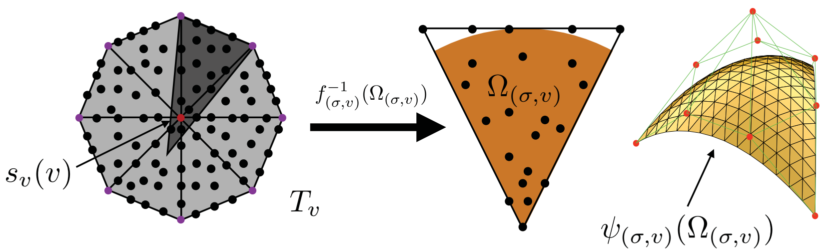
- We use Floater's parametrization to build the map for each "curved" triangle.



Adaptive Fitting

Create S from $\tilde{S}_T$

- For each triangle in $st(v, \tilde{S}_T)$, compute the shape function $\psi_{(\sigma,v)}$.



Adaptive Fitting

Create S from $\tilde{S}_T$

- Control points of $\psi_{(\sigma,v)}$ are computed by a least squares procedure.

- But, this time, the sample points are the vertices of S_T that correspond to the points in T_v through Floater's parametrization!

Adaptive Fitting

Create S from $\tilde{S}_T$

- For each point p in T_v, we compute the **approximation error**,

$$\left\| q - \psi_{(\sigma,v)(p)} \right\|,$$

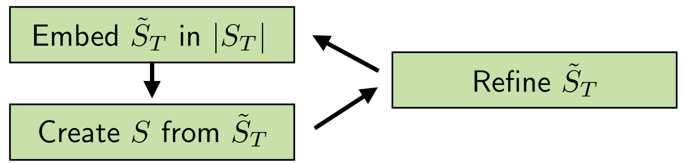
  where q is the vertex of S_T corresponding to p through Floater's parametrization.

- If the above error is smaller than the given number ϵ, we keep computing $\psi_{(\tau,u)}$, for all pairs $(\tau, u) \in I$. Otherwise, we stop this process and go to the refinement step.

Adaptive Fitting

Refine $\tilde{S}_T$

- We locally refine $\tilde{S}_T$ using the stellar operations and the 4-8 refinement, and then embed the resulting $\tilde{S}_T$ in $|S_T|$ again.



Discrete Geodesics

- **Locally Shortest Geodesic**:
  A curve joining two points, A and B, on a polyhedral surface. It is a local minimum of the length functional.

- **Straighest Geodesic**:
  A curve beginning at point A and moving in the direction of the tangent vector. It has zero *discrete geodesic curvature* everywhere.

Discrete Geodesics

Locally shortest geodesics:

Exact algorithms:

- Mitchell, Mount, and Papadimitriou (1987)
- Chen and Han (1996)
- Kapoor (1999)
- Surazhsky, Surazhsky, Kirsanov, Gortler, and Hoppe (2005)

Discrete Geodesics

Locally shortest geodesics:

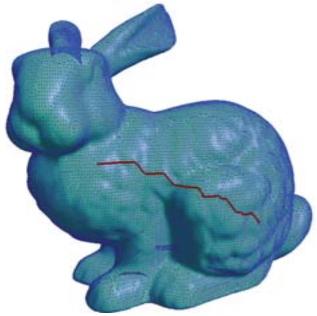
Approximate algorithms:

- Kimmel and Sethian (1998)
- Martínez, Velho, and Carvalho (2004)
- Surazhsky, Surazhsky, Kirsanov, Gortler, and Hoppe (2005)

Discrete Geodesics

A Two-Step Algorithm:

Step 1:
Find an initial curve joining A and B.



Discrete Geodesics

A Two-Step Algorithm:

Step 2:
Iteratively modify the position of each curve vertex.

Discrete Geodesics

Step 1:
Find an initial curve joining A and B.

- Fast Marching Method
- Define a distance function at the vertices, $d(v) = \mathrm{dist}(A, V)$, using an approximation of the eikonal equation

$$|\nabla d| = 1\,.$$

Discrete Geodesics

Step 1:
Find an initial curve joining A and B.

- Back-integrate the differential equation:

$$\begin{cases} \frac{d\Gamma_0}{ds}(s) = -\nabla d(\Gamma_0(s)) \\ \Gamma_0(0) = B\,. \end{cases}$$

Discrete Geodesics

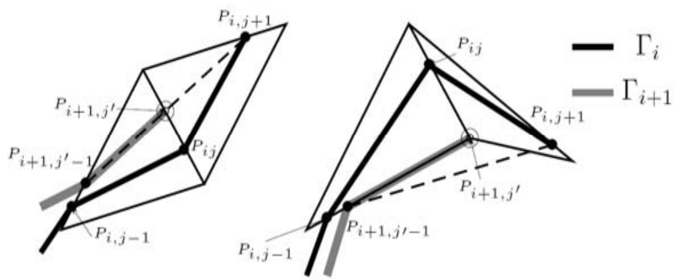
Step 2:
Iteratively modify the position of each curve vertex.

- Given a curve Γ_i, we want to get a shorter curve, Γ_{i+1}, with the same endpoints.
  - a geodesic should be a line segment in the interior of a face;
  - the curve will be a polygonal with nodes belonging to the edges of the mesh;
  - the algorithm will correct the position of the curve nodes;
  - distinct behavior for "edge nodes" and "vertex nodes".

Discrete Geodesics

Step 2:
Iteratively modify the position of each curve vertex.

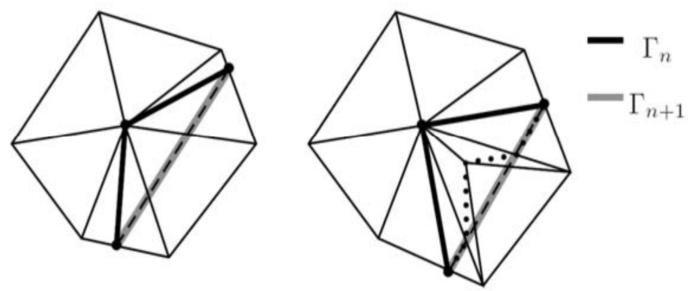
Edges nodes:



Discrete Geodesics

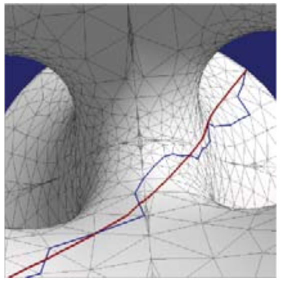
Step 2:
Iteratively modify the position of each curve vertex.

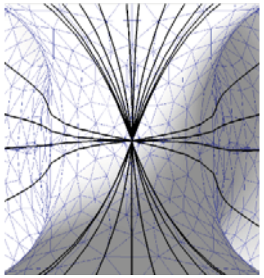
Vertex nodes:



Discrete Geodesics

Examples:

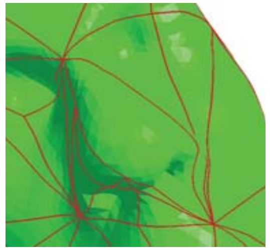




Discrete Geodesics

Adaptive Fitting:

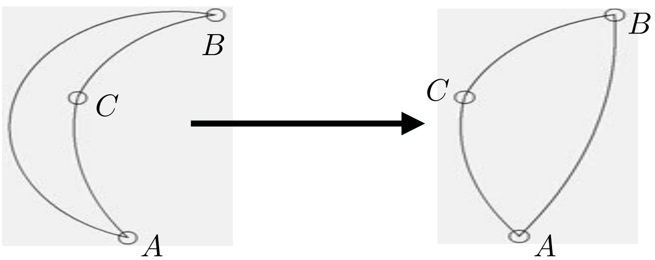
When defining geodesic triangles, we can violate the manifold property of the geodesic mesh, as illustrated by the figure below:



Discrete Geodesics

Solution:

Compute the geodesic curve AB as the geodesic resulting from using the concatenation of geodesics AC and CB as initial approximation:



Conclusions

- Adaptive fitting pipeline is not new, but its elements are.
- This is the “real deal” when it comes to comparisons between smooth surfaces and very dense polygonal meshes.
- Implementation of the adaptive fitting is still under development.
- More specifically, the refinement step has not been completed.

Applications of Manifolds and Research Challenges

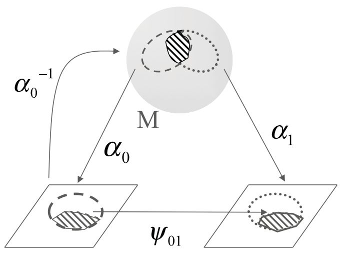
Luiz Velho
IMPA

Outline

- Concepts
- Illumination
- Appearance
- Simulation
- Faces
- Manifold Learning
- Wrap-up

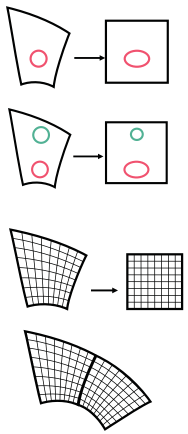
Manifolds & Parametrization

- Two Points of View
  - Functions on surfaces
  - Functions defining surfaces



Desirable Properties

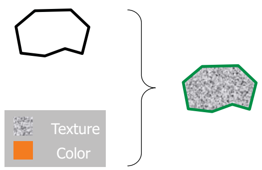
- Minimal Distortion
  - Angle
  - Area
- Smoothness
  - Differentiability
  - Continuity



Graphical Objects

- Shape U
  - Topology (*domain*)
    - Abstract Manifold
  - Geometry (*function*)
    - Embedding
- Attributes f
  - Functions (*co-domain*)

$$O = (U,\ f)$$



G.O. Manifold Setting

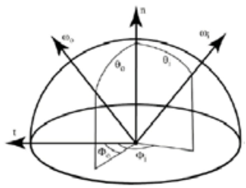
- Canonical Surfaces
  - Fixed Shape (defined *apriori*)
  - Variable Functions (complex)
    - *ex: Sphere*
- Arbitrary Surfaces
  - Complex Shape
  - Computation on Surfaces (attributes)
  - Building / Transforming (shape)
    - *ex: Triangle Meshes*

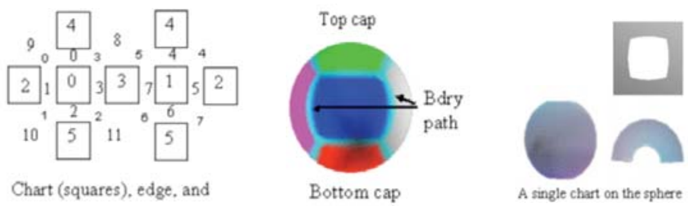
Applications

- Illumination
  - Canonical Manifold + Functions
- Appearance and Simulation
  - Pseudo-Manifold + Attributes
- Faces
  - Manifold + Geometric Deformation
- Surface Reconstruction
  - Pseudo-Manifold / Topology Estimation

Illumination

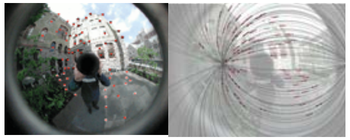
- Functions on the Sphere
  - Light Fields / BRDFs
- Applications
  - Capture / Synthesis
- Construction [Grimm 2002]



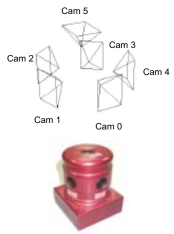


Omnidirectional Images

- Panoramic Cameras
  - Processing



- Multi-Camera Assembly
  - Stitching / Blending

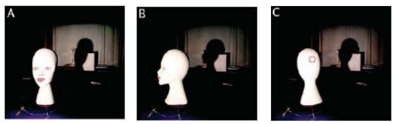


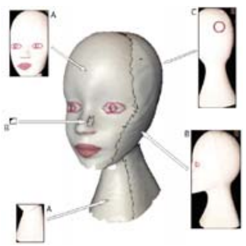
Illumination Maps

- Environment Maps
  - Area Sampling
- Light Maps
  - Stratification

Surface Properties

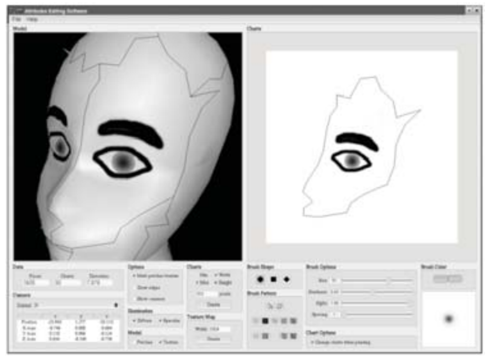
- Texture Atlas
  - Albedo
  - Normal Field
- Building from Images
  - Projective Map





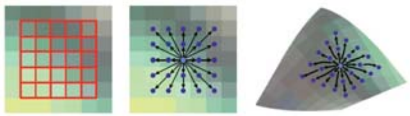
Painting

- Color
- Normals



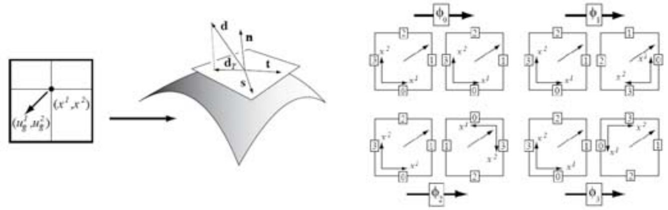
Texture Synthesis

- Stationary / Quasi Stationary



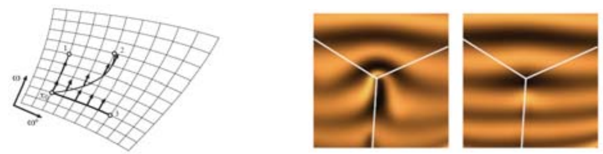
Simulation

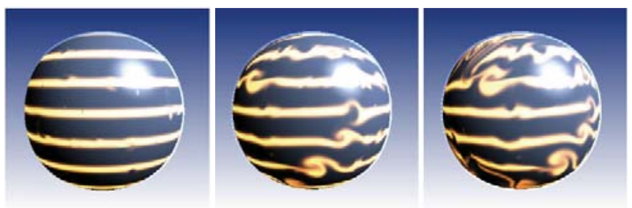
- Solving Equations on Manifolds
  - Surface Points
  - Local Neighborhoods



Fluids

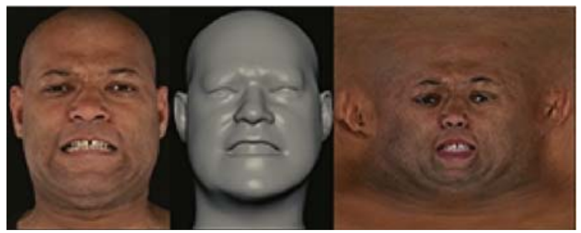
- Vector Fields on Surfaces





Faces

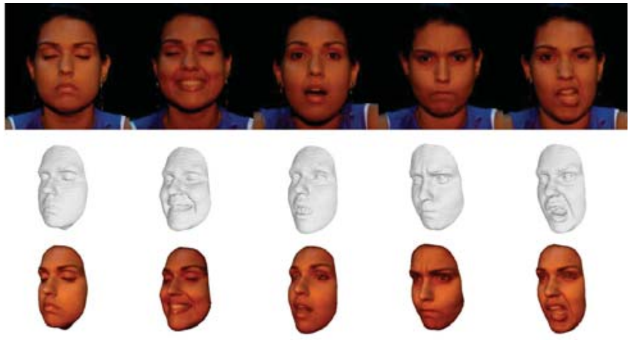
- Geometry + Appearance



[G. Borshukov et al SIGGRAPH 2003]

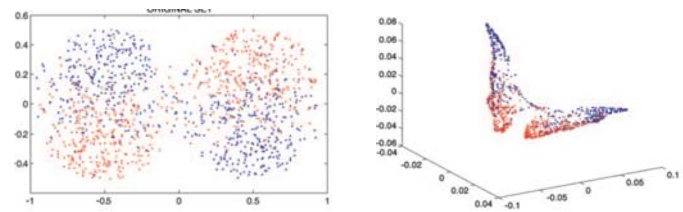
Facial Expressions

- Deformations



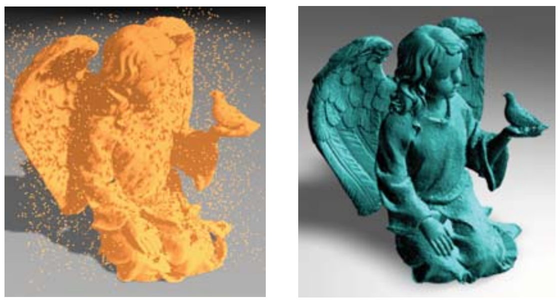
Manifold Learning

- Estimate from Data Samples
  - Topology
  - Geometry



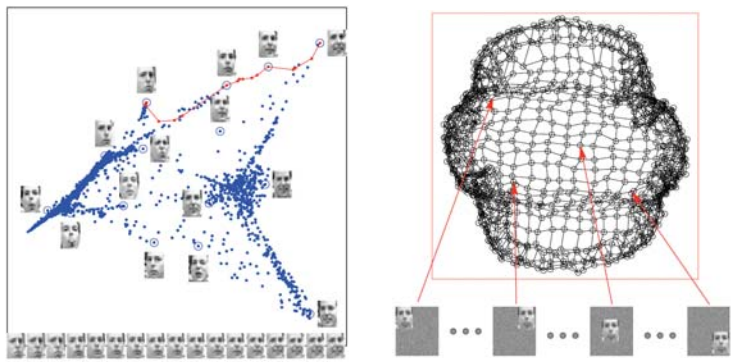
Surfaces

- Point Sets



N-Dimensional Case

- ex: Facial Expressions



Challenges

- Multi-Resolution
  - Hierarchical Atlas
  - Dynamic Setting
- API
  - Intuitive
  - General

Questions ?