Efficient Computation of Global Illumination for Image Synthesis

Anamaria Gomide, Danillo Roberto Pereira, Jorge Stolfi
Institute of Computing – University of Campinas (UNICAMP)
13083-970 , Campinas, SP, Brazil
anamaria@ic.unicamp.br,danillo.pereira@students.ic.unicamp.br,stolfi@ic.unicamp.br

Abstract

We are exploring various functional approximation schemes (such as radial bases and Shepard's interpolation) for precomputed radiance transfer (PRT), a finite element approach for real-time radiosity of complex scenes.

1. Introduction

Radiosity [1] is a general method for realistic rendering that uses finite element modeling to solve the Kajiya's *rendering equation* [3]. In this formulation, the light flow in the scene is found by solving a large system of linear equations $(I - R)\lambda = \epsilon$, where the vector ϵ gives the spontaneous light emission and λ gives the total emission (spontaneous plus scattered) of each scene element.

Implementation of the radiosity method requires choosing a finite element scheme to model the light flow. The goal of our project is to compare various schemes, such as radial bases [2] and Shepard's interpolation [5] basis - especially for use in real-time animation through *precomputed radiance transfer* [4, 6].

2. The rendering equation

The rendering equation can be written as $L = E + \mathcal{R}L$, where:

• $L(x, \vec{u})$ is the (unknown) *radiance function*, the total amount light emitted or scattered by the scene near the surface point x along directions near the unit vector $\vec{u}$;

• $E(x, \vec{u})$ is the *spontaneous emission function*, the amount of light emitted by the scene near x and along $\vec{u}$;

• $\mathcal{R}$ is the *light transfer operator*, that represents the transport and interaction of the light with the scene's objects. Its effect on an arbitrary function $F(x, \vec{u})$ is

$$(\mathcal{R}F)(x, \vec{u}) = \int_{\mathcal{S}} \rho(x, \vec{v}, \vec{u}) F(x', -\vec{v}) G(x, x', \vec{n}(x)) d\vec{v} \quad (1)$$

• $\mathcal{S}$ is the set of all directions (i.e. the unit sphere).

• $x' = x^*(x, \vec{v})$ is the first point on the scene's surface found by a ray that leaves x towards $\vec{v}$.

• $\rho(x, \vec{v}, \vec{u})$ is the *scattering coefficient*, the fraction of the incident light at the point x, coming from the direction $\vec{v}$, that is re-emitted along directions near $\vec{u}$;

• $G(x, x', \vec{n}(x))$ is the *geometric factor* that depends on the orientation of the surface at x.

In favorable circumstances, the solution of the rendering equation is
$L = (\mathcal{I} - \mathcal{R})^{-1}E$, where $\mathcal{I}$ is the identity operator. The *rendering operator* $(\mathcal{I} - \mathcal{R})^{-1}$ can be computed by Neumann's formula

$$(\mathcal{I} - \mathcal{R})^{-1} = (\mathcal{I} + \mathcal{R} + \mathcal{R}^2 + \mathcal{R}^3 + \cdots) \quad (2)$$

Each term $\mathcal{R}^k$ accounts for light that interacted k times with the surface of the scene before being observed.

3. Finite element radiosity

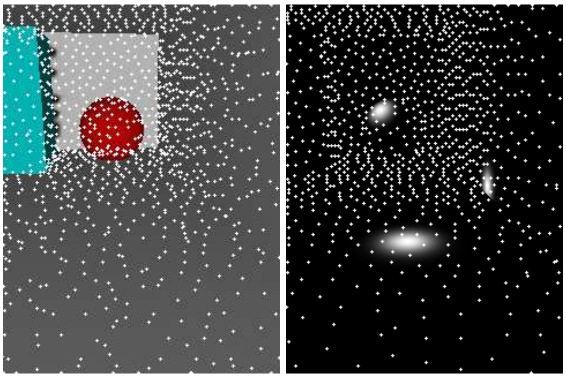
We use the term *site* to mean a pair $p = (\dot{p}, \vec{p})$ where $\dot{p}$ is a point on the scene's surface and $\vec{p}$ is the corresponding unit normal vector. For Lambertian radiosity, a *finite element* is a function ϕ defined on the scene's surface sites, such that $\phi(p)$ is nonzero only for a relatively small and compact set of sites (the *support* of ϕ).

In *point-based finite element methods*, the basis ϕ is defined by selecting a number of *sampling sites* $P = \{p_1, p_2, ...p_n\}$ on the scene's surface (see Figure 1(a)) and then choosing for each p_i a corresponding finite element ϕ_i that has p_i as its centroid (see Figure 1(b)).

3.1 Basis functions

In a *radial basis* each element ϕ_i is derived from fixed *mother function* ϕ by the formula $\phi_i(p) = \Phi\left(||p, p_i||/\alpha_i\right)$; where $||p, q||$, denotes the distance between the sites p and q, and α_i is a parameter that determines the "mean radius" of the element. See figure 2(a).

In *Shepard's basis* [5], each element is

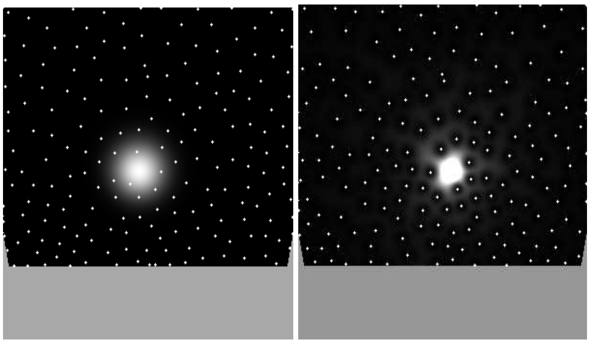


(a) (b)

Figure 1. The point-based FE method.

$$\phi_i(p) = \frac{w(p, p_i)}{\sum_{j=1}^{n} w(p, p_j)}$$

Here $w(p, q)$ is a non-negative function that tends to infinity when p tends to qe. g. $1/||p, q||$. See figure 2(b).



(a) (b)

Figure 2. Radial and Shepard elements.

4. Discretizing the rendering equation

In finite element radiosity, the rendering equation becomes a linear equation system $\lambda = \epsilon + R\lambda$, where $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)$ are the coefficients of L in the chosen basis, and $\epsilon = (\epsilon_1, \epsilon_2, ..., \epsilon_n)$ are the coefficients of E. Each element R_{jk} represents the influence of the light emitted by ϕ_k on the light scattered by ϕ_j. It is determined by the formula

$$R_{jk} = G(\dot{p}_k, \dot{p}_j, \vec{p}_j)\, \rho(p_j)\, V(\dot{p}_k, \dot{p}_j) \tag{3}$$

Once the matrix R is available, the coefficients λ_i of the radiance function L can be computed by seting $\lambda \leftarrow (0, ..., 0)$ and then iterating $\lambda \leftarrow \epsilon + R\lambda$ until convergence.

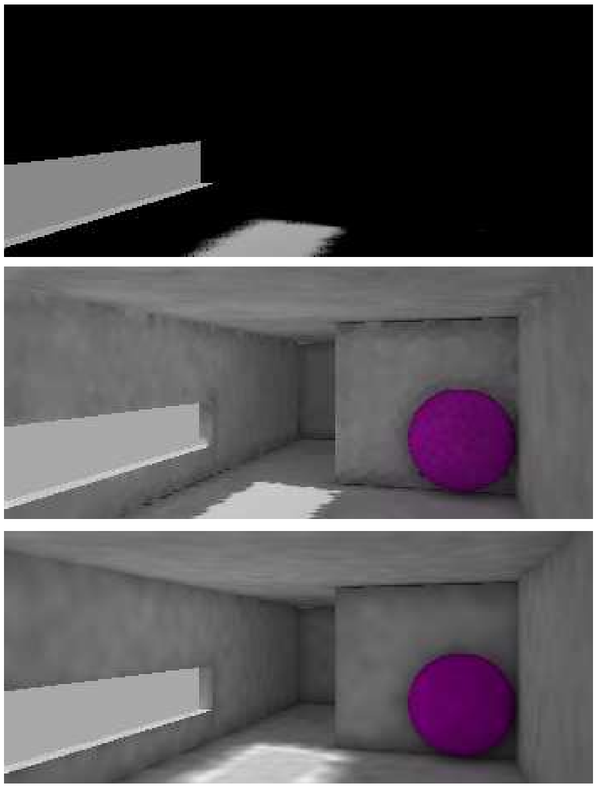


Figure 3. Rendered scenes.

5. Results

Figure 3 shows a test scene rendered without radiosity (top) and with ten radiosity iterations, using Shepard interpolation (middle) and a radial basis (bottom).

References

[1] M. F. Cohen and J. R. Wallace. *Radiosity and Realistic Image Synthesis*. Academic Press, 1993.

[2] M. S. Floater and A. Iske. Multistep scattered data interpolation using compactly supported radial basis functions. *J. Comput. Appl. Math.*, 73(1-2):65–78, 1996.

[3] J. T. Kajiya. The rendering equation. In *Proc. SIGGRAPH 1986*, pages 143–150, 1986.

[4] J. Lehtinen, M. Zwicker, J. Kontkanen, E. Turquin, F. Sillion, and T. Aila. Meshless finite elements for hierarchical global illumination. Technical Report TML-B7, Helsinki Univ. of Technology, 2007.

[5] D. Shepard. A two-dimensional interpolation function for irregularly-spaced data. In *Proc. 23rd ACM National Conference*, pages 517–524, 1968.

[6] P. P. Sloan, J. Kautz, and J. Snyder. Precomputed radiance transfer for real-time rendering in dynamic, low-frequency lighting environments. *ACM Trans. on Comp. Graphics*, 21(3):527–536, 2002.