Surface Equalization

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Abstract

Subdivision is a method to create smooth surfaces through a refinable schema of polygonal or triangular meshes. From a mathematical point of view, this technique is an application of a second generation wavelet analysis. At the same time, this analysis is based on the lifting schema and does not use any frequency criterion to define scaling functions or wavelets. This simplifies computational cost. Yet, a frequency analysis applied to a polygonal mesh, can provide an intuitive method to modify surface characteristics by creating an analogy to one-dimensional sound equalization. These foundations can be useful not only as a surface generation tool, but they are also naturally associated to finite element techniques and can be applied to a wide variety of simulation problems.

Keywords: subdivision surfaces, frequency analysis, Fourier, multirresolution, equalization, radiosity

1. Introduction

Subdivision techniques take a control mesh or polyhedron and add new polygons to it by refining its current faces. These new vertices are then perturbed according to some stencil rule, adding details and, in most cases, increasing surfaces's smoothness. Subdivision techniques only need a good and fast structure to preserve adjacencies between the polygons that approximate a surface. These polygons can be triangles, but many subdivision schemas can be adapted to other kinds of polygons [3].

As NURBS and Splines [8], subdivision techniques have a very desirable property: perturbation of one vertex can be made locally. By using subdivision and an appropriate stencil rule (such as Loop's [5]), more triangles can be added to an arbitrary mesh preserving continuity. This gives a nice tool to generate and manipulate smooth arbitrary surfaces, topologically equivalent to a basic shape.

An example of this process is shown in figure 1.

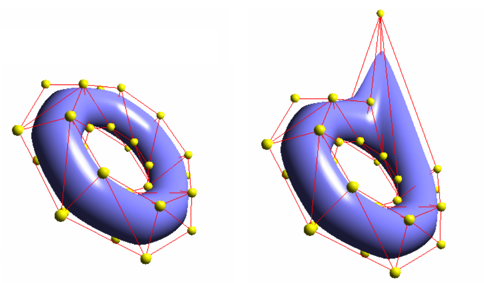

Figure 1. A deformed torus. Yellow spheres represent control points.

2. Equalizing Images and Polyhedra

From a mathematical point of view, equalizing an image is equivalent to applying an operator to a bidimensional array of numbers (a matrix). A frequency interpretation of this process is given by discrete bidimensional Fourier basis, which can be considered as a set of eigenvectors of a given operator [4]. From this discrete approach, it is relatively easy to define a multirresolution analysis (defined in [7]) for vector spaces of matrices, as an increasing set of finite-dimensional subspaces generated by a well chosen basis. These bases are called wavelets. Interpreting these subspaces and projecting the original matrix on them, we get a criterion to select coefficients in a basis expansion. As this criterion is based on a frequency filtering concept, we have a multirresolutive image equalizer. This is shown in figure 2.

Equalizing images is a somehow straightforward task because they can be sampled as regular grids and matrices. Extending this concept to general polyhedra is not trivial because they are not necessarily regular and cannot be generally represented by matrices.

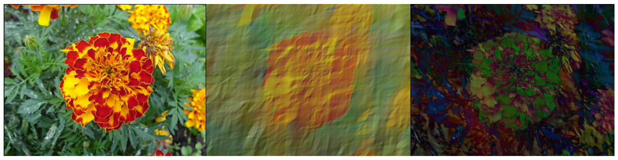

Figure 2. Original image. Low-pass filter and high-pass filter.

Nevertheless, in 1997, it was shown that subdivision techniques are a particular case of general multirresolution analysis on arbitrary topologies [6]. The method used to build wavelets with this degree of freedom is an application of the more general lifting method for second generation wavelets [9]. This technique has no need to use any frequency criterion to build a semiorthogonal multirresolution analysis. It is generally used to smooth a surface (which is equivalent to apply a low-pass filter).

3. Vibration Modes of a Polyhedron

In our research, we are trying to find a frequency criterion to control de degree of smoothness produced through a subdivision schema. As described above, Fourier analysis on matrices can be considered as a method to exploit conceptually an eigenvector basis of certain matrices. As described in [10], eigenvectors of a connectivity matrix can be considered the vibration modes of a polyhedron. Two characteristics of this technique is that the number of faces remains unchanged and it is applied globally. So, being able to link this approach with the lifting schema, could produce an equalizing subdivision stencil that could locally add smooth details or sharp details to a polyhedron. This could be done according to a criterion defined by the user. A global sharp surface subdivision process is shown in figure 3. As we pretend to have a better equalizing user's control of this, the figure is shown only for illustrative purposes.

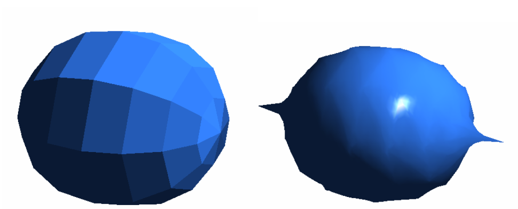

Figure 3. A non-smooth subdivision surface

A lifting schema based on frequency can be computationally slower than Lounsberry's [6], as it implies to get eigenvalues of large matrices (as large as the number of vertices of the polyhedron). Our hypothesis is that eigenvalues of a polyhedron's connectivity matrix should be naturally associated to its degree of smoothness in the lifting schema. We believe that the method described in [10] can be extended into an intuitive frequency-based method to design second generation wavelets. Equalization to design wavelets can be used directly on regular grids (such as figure 2) in a somehow straight application of classical wavelet theory. Yet, as far as we know, there is no general method to use the lifting schema with a frequency criterion to create a subdivision technique based on Fourier coefficients.

4. Future Research

As the lifting schema and subdivision methods can be used to define functions on a polyhedron, the techniques described above can be used to define an adaptive finite element set [1], which can be used in simulation problems such as radiosity [2]. It gives an intuitive method to construct wavelets based on discrete digital filters. This can be useful to naturally deal with discontinuities of functions defined on arbitrary topologies.

References

[1] S. C. Brenner and L. R. Scott, *The mathematical theory of finite element methods*, Springer-Verlag, Ed. Springer, 2000.

[2] M. F. Cohen and J. R. Wallace, *Radiosity and realistic image synthesis*, H. Brace and Company, Eds. Academic Press Professional, 1993.

[3] P. S. Denis Zorin, "Subdivision for modeling and animation," *Siggraph 2000 Course Notes*, vol. Dowonloadable from http://mrl.nyu.edu/ dzorin/sig00course/, 2000.

[4] M. Frazier, *An introduction to wavelets through linear algebra*, Springer-Verlag, Ed. Springer, 1999.

[5] C. T. Loop, "Smooth subdivision surfaces based on triangles," Ph.D. dissertation, University of Utah, 1987.

[6] M. Lounsberry, T. D. DeRose, and J. Warren, "Multirresolution analysis for surfaces of arbitrary topologial type," *ACM Transactions of graphics*, vol. 16, issue 1,, pp. 34–73, Juanuary, 1997.

[7] S. Mallat, *A wavelet tour of signal processing*, P. Ecole Poolytechnic, Ed. Academic Press, 1999.

[8] D. Solomon, *Curves and surfaces for computer graphics*, S. S.-B. Media, Ed. Springer, 2006.

[9] W. Sweldens, "The lifting schema: A construction of second generation wavelets," *Mathematics Subject Classification SIAM Journal on Mathematical Analysis*, vol. 42C15, 1995.

[10] G. Taubin, "A signal processing approach to fair surface design," *International Conference on Computer Graphics and Interactive Techniques Proceedings of the 22nd annual conference on Computer graphics and interactive techniques*, vol. IBM T.J. Watson Research Center, P.O.Box 704, Yorktown Heights, NY, pp. pp: 351 – 358, 1995.